Lecture 5

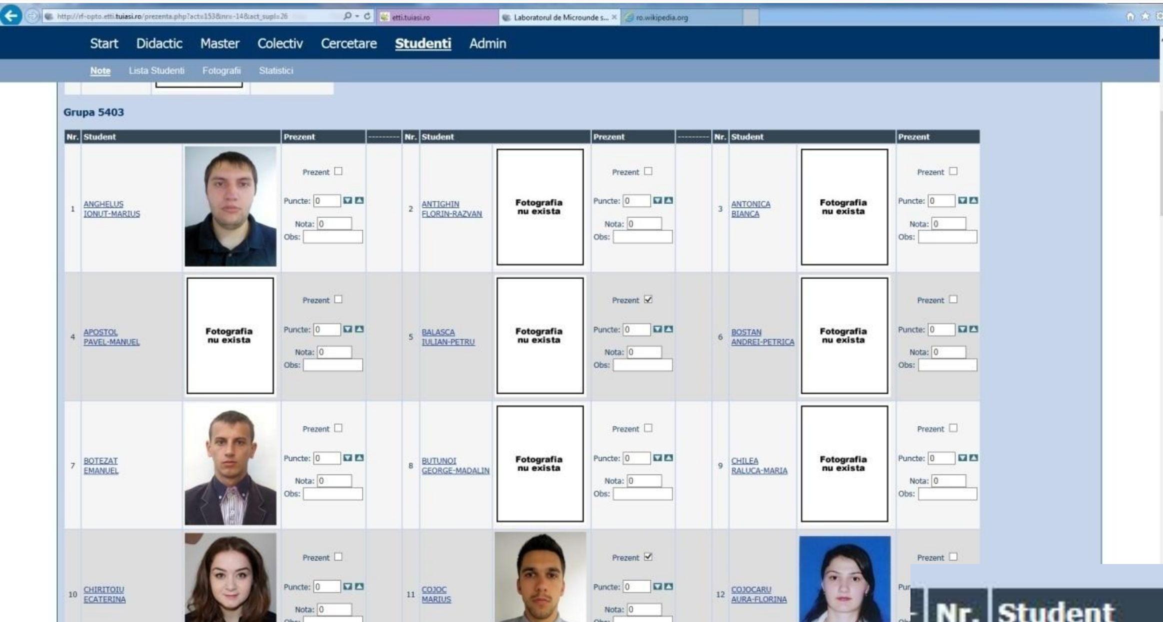
2017/2018

Microwave Devices and Circuits for Radiocommunications

Materials

- RF-OPTO
  - **http://rf-opto.etti.tuiasi.ro**
- **David Pozar, "Microwave Engineering"**, Wiley; 4th edition , 2011
  - 1 exam problem ← Pozar
- Photos
  - sent by email: rdamian@etti.tuiasi.ro
  - used at lectures/laboratory

Photos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Access

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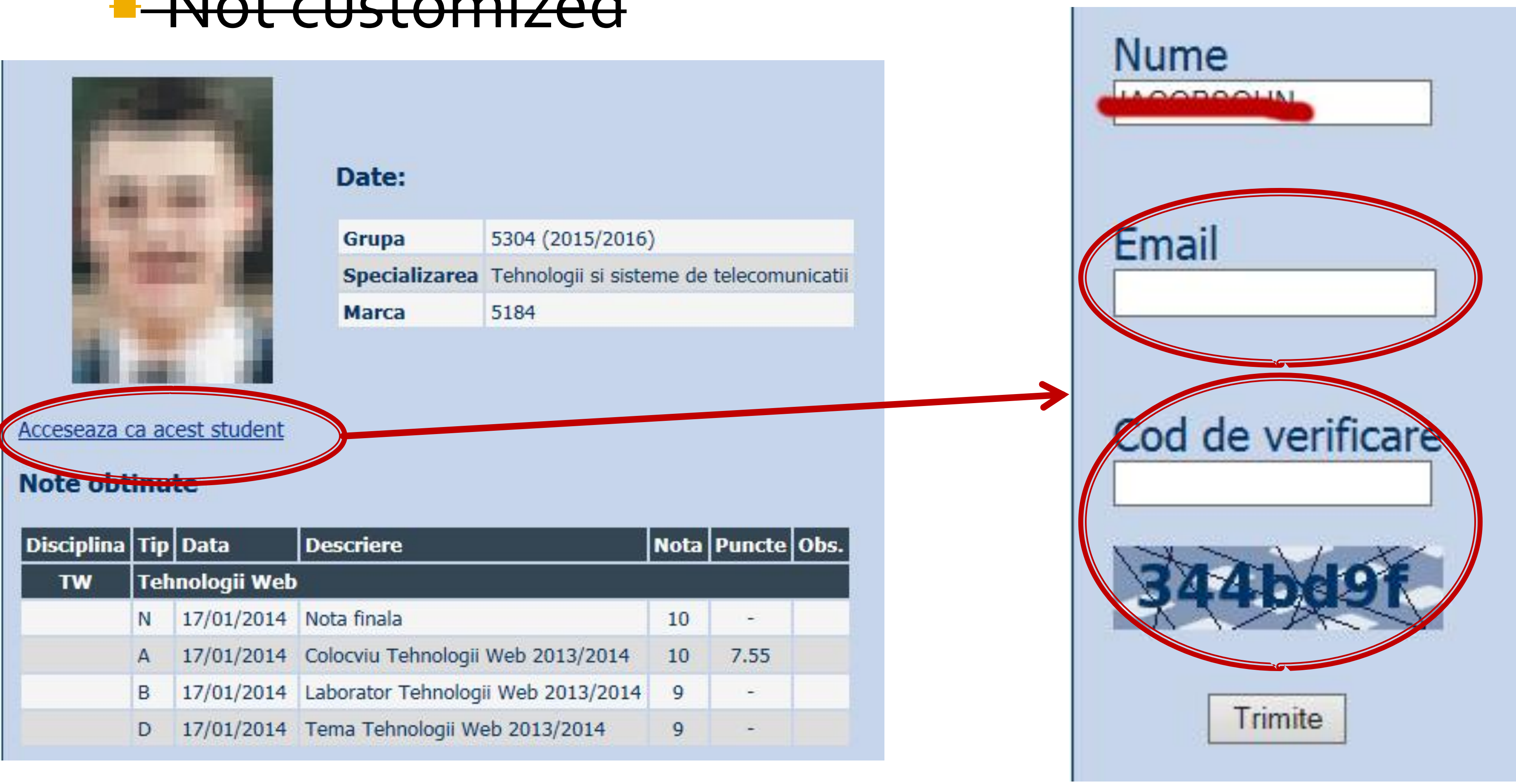
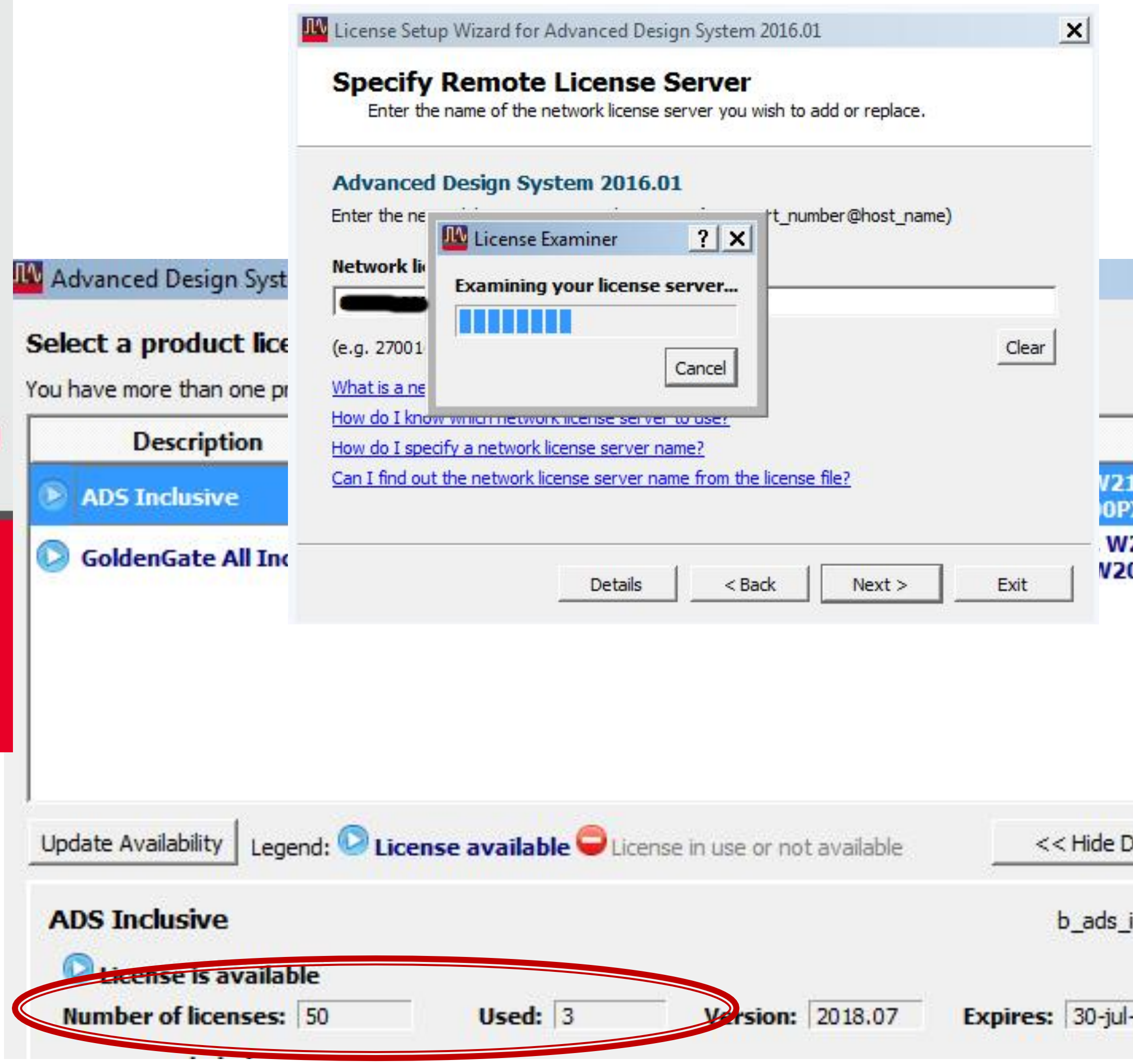


Software

- ADS 2016
- EmPro 2015
- based on IP from outside university or campus

Software


Advanced Design System
Premier High-Frequency and High Speed Design Platform
2016.01
KEYSIGHT TECHNOLOGIES
© Keysight Technologies 1985-2016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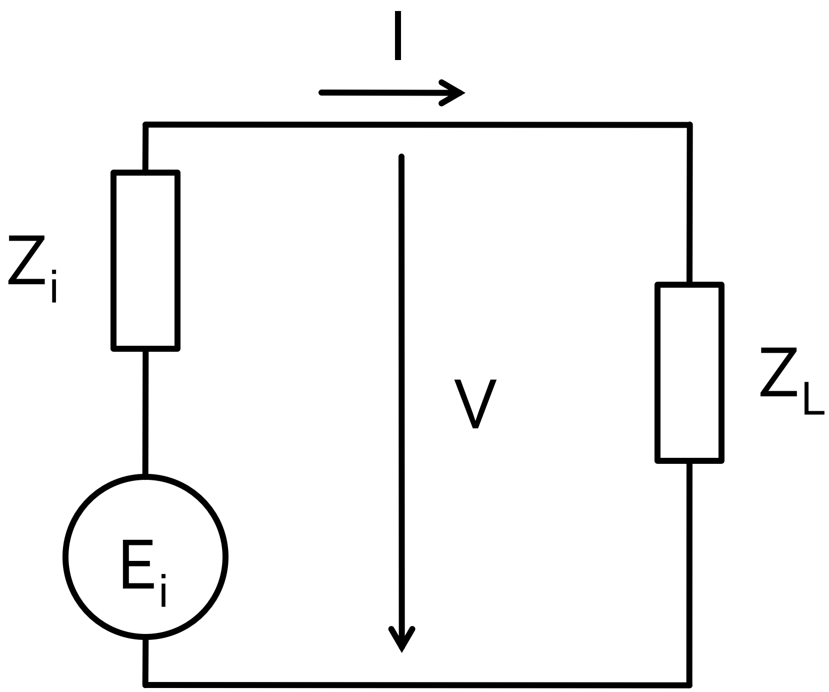
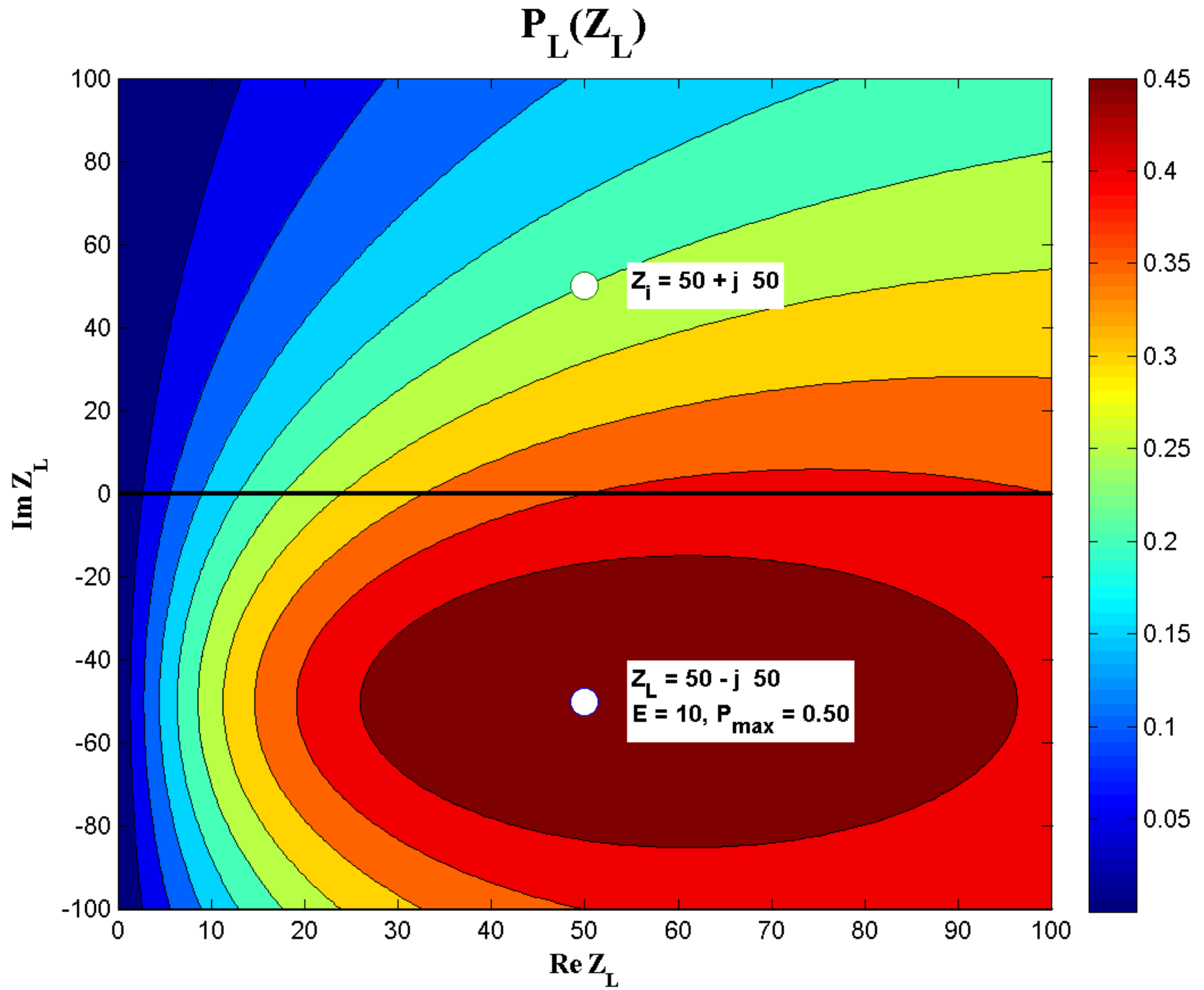
Matching

- Source **matched** to load ?



- impedance values ?
- existence of reflections ?

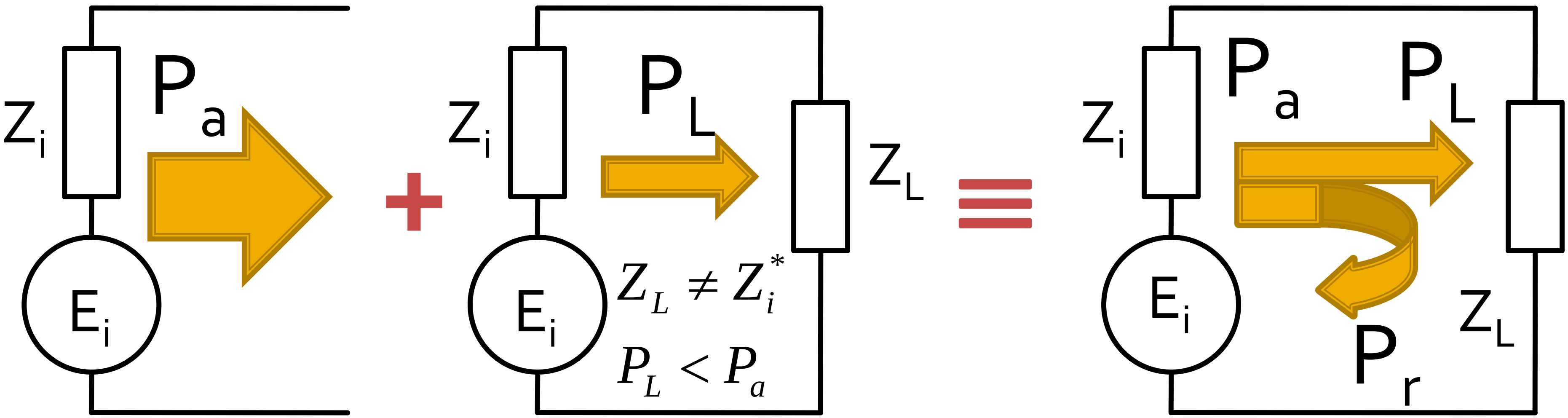
Matching, example



$$Z_L = Z_i^*$$

$$\Gamma_L = \Gamma_i^*$$

Reflection and power / Model

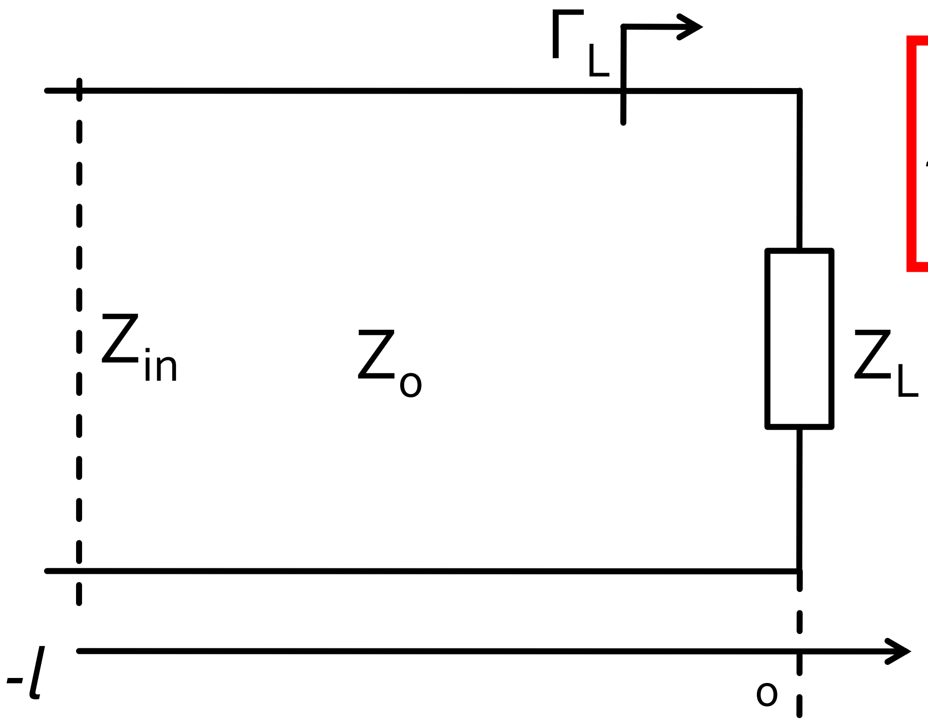


- The source has the ability to sent to the load a certain maximum power (available power) P_a
- For a particular load the power sent to the load is less than the maximum (mismatch) $P_L < P_a$
- The phenomenon is **"as if"** (model) part of the input power is reflected $P_r = P_a - P_L$
- The power is a **scalar !**

TEM transmission lines

The lossless line

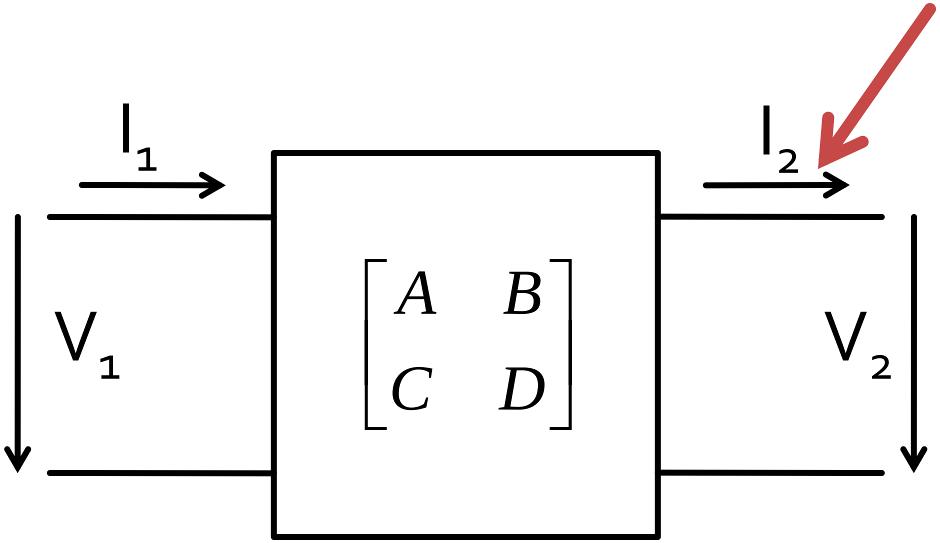
- input impedance of a length ***l*** of transmission line with characteristic impedance $\boldsymbol{Z_o}$, loaded with an arbitrary impedance $\boldsymbol{Z_L}$



$$Z_{in} = Z_0 \cdot \frac{Z_L + j \cdot Z_0 \cdot \tan \beta \cdot l}{Z_0 + j \cdot Z_L \cdot \tan \beta \cdot l}$$

Microwave Network Analysis

ABCD (transmission) matrix



$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \cdot \begin{bmatrix} V_2 \\ I_2 \end{bmatrix}$$

$$V_1 = A \cdot V_2 + B \cdot I_2$$

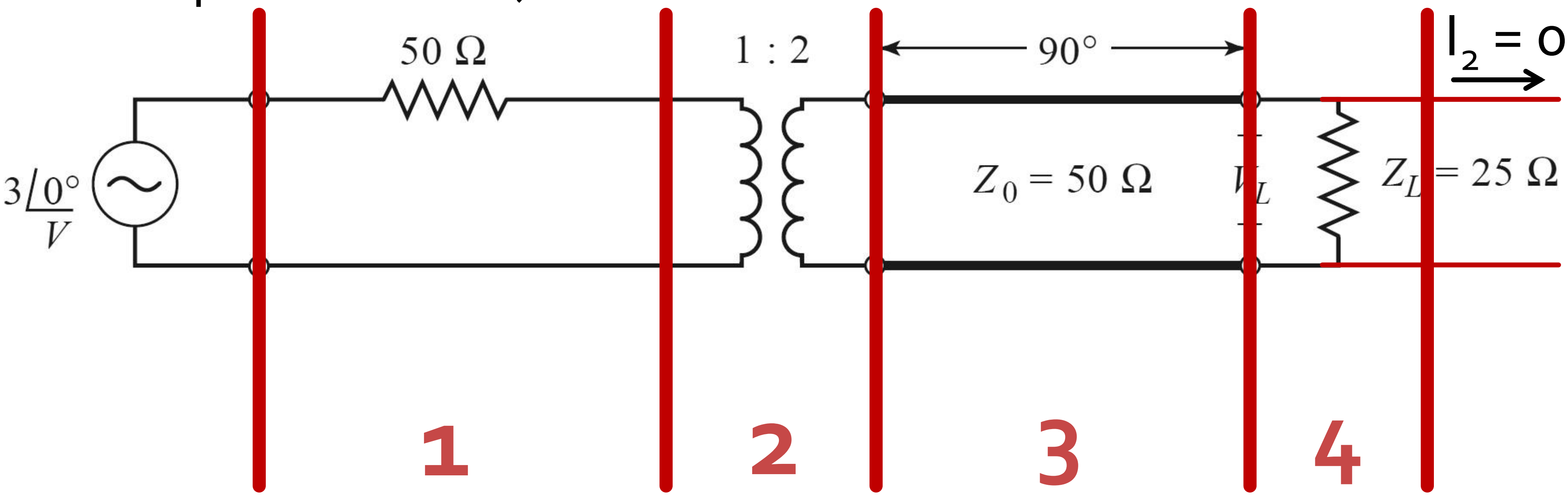
$$I_1 = C \cdot V_2 + D \cdot I_2$$

$$\begin{bmatrix} V_2 \\ I_2 \end{bmatrix} = \frac{1}{A \cdot D - B \cdot C} \cdot \begin{bmatrix} D & -B \\ -C & A \end{bmatrix} \cdot \begin{bmatrix} V_1 \\ I_1 \end{bmatrix}$$

$$A = \left.\frac{V_1}{V_2}\right|_{I_2=0} \qquad B = \left.\frac{V_1}{I_2}\right|_{V_2=0} \qquad C = \left.\frac{I_1}{V_2}\right|_{I_2=0} \qquad D = \left.\frac{I_1}{I_2}\right|_{V_2=0}$$

Example for ABCD matrix

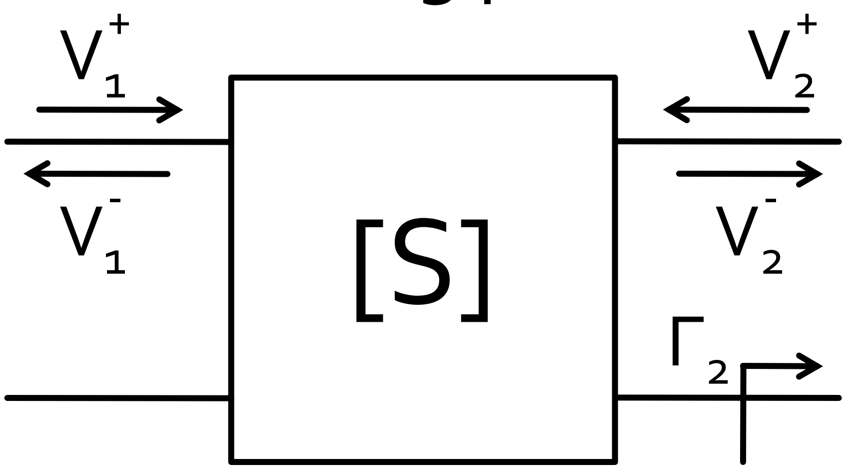
- We break the circuit in elementary sections
- Sources are left outside
- If necessary, input and output ports are created (and left open-circuited)



$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = M_1 \cdot M_2 \cdot M_3 \cdot M_4 \qquad V_1 = A \cdot V_2 + B \cdot I_2 \Big|_{I_2=0} \qquad V = A \cdot V_L \rightarrow V_L = \frac{V}{A}$$

Scattering matrix – S

- Scattering parameters



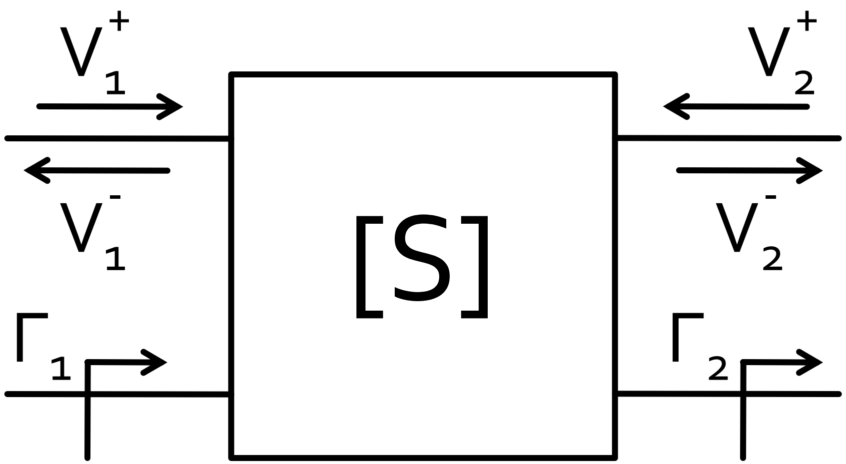
$$\begin{bmatrix} V_1^- \\ V_2^- \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \cdot \begin{bmatrix} V_1^+ \\ V_2^+ \end{bmatrix}$$

$$S_{11} = \left. \frac{V_1^-}{V_1^+} \right|_{V_2^+=0} \qquad S_{21} = \left. \frac{V_2^-}{V_1^+} \right|_{V_2^+=0}$$

- $V_2^+ = 0$ meaning: port 2 is terminated in matched load to avoid reflections towards the port

$$\Gamma_2 = 0 \rightarrow V_2^+ = 0$$

Scattering matrix – S



$$\begin{bmatrix} V_1^- \\ V_2^- \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \cdot \begin{bmatrix} V_1^+ \\ V_2^+ \end{bmatrix}$$

$$S_{11} = \left.\frac{V_1^-}{V_1^+}\right|_{V_2^+=0} = \Gamma_1\big|_{\Gamma_2=0}$$

$$S_{21} = \left.\frac{V_2^-}{V_1^+}\right|_{V_2^+=0} = T_{21}\big|_{\Gamma_2=0}$$

- S11 is the reflection coefficient seen looking into port ***1*** when port ***2*** is terminated in matched load
- S21 is the transmission coefficient from port ***1*** (**second index**) to port ***2*** (**first index**) when port ***2*** is terminated in matched load

Generalized Scattering Parameters

- We define the power wave amplitudes a and b

$$a = \frac{V + Z_R \cdot I}{2 \cdot \sqrt{R_R}}$$ the incident power wave

$$b = \frac{V - Z_R^* \cdot I}{2 \cdot \sqrt{R_R}}$$ the reflected power wave

$$Z_R = R_R + j \cdot X_R$$

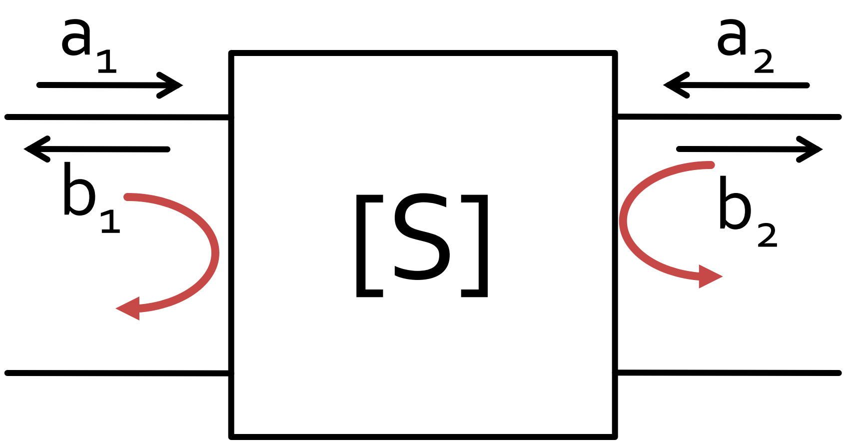
Any complex impedance, named reference impedance

- Total voltage and current in terms of the power wave amplitudes

$$V = \frac{Z_R^* \cdot a + Z_R \cdot b}{\sqrt{R_R}}$$

$$I = \frac{a - b}{\sqrt{R_R}}$$

Scattering matrix – S

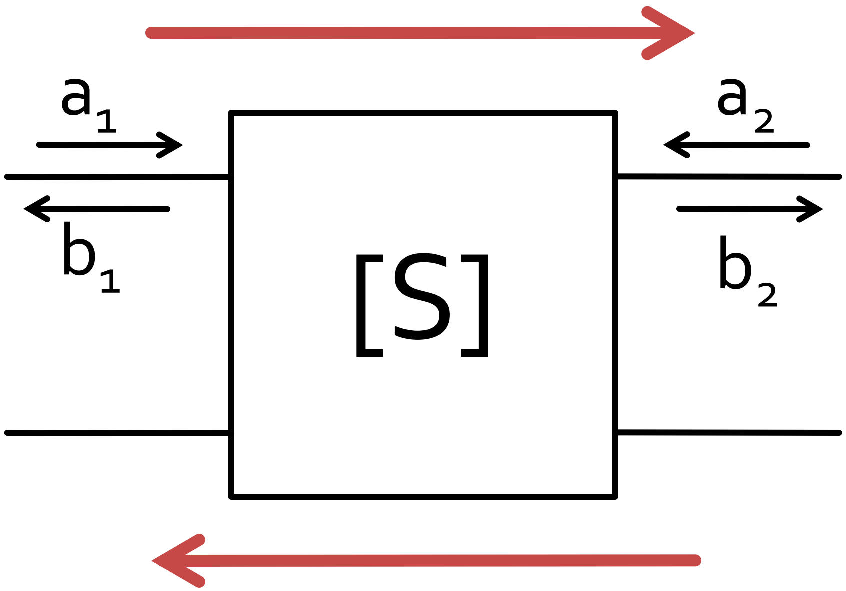


$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \cdot \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$

$$S_{11} = \left. \frac{b_1}{a_1} \right|_{a_2=0} \qquad S_{22} = \left. \frac{b_2}{a_2} \right|_{a_1=0}$$

- S_{11} and S_{22} are reflection coefficients at ports 1 and 2 when the other port is mathed

Scattering matrix – S

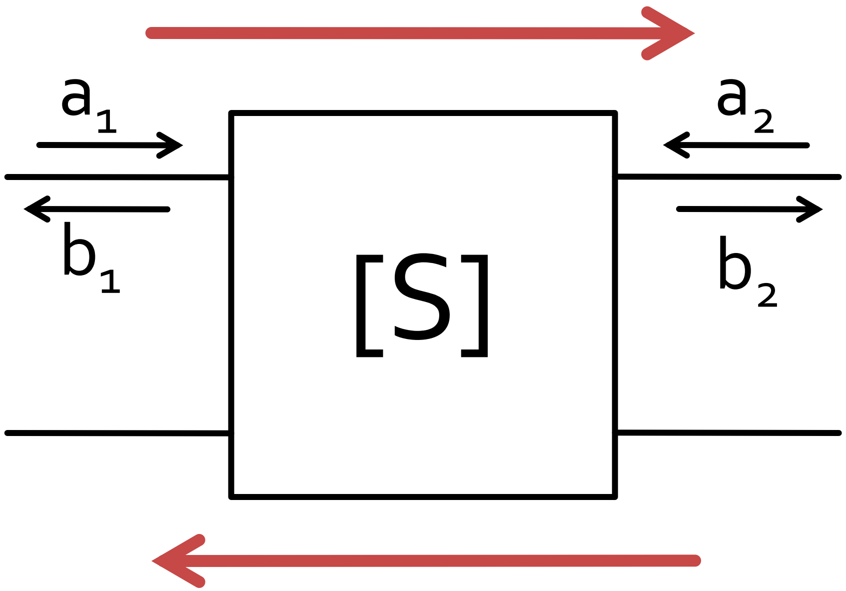


$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \cdot \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$

$$S_{21} = \left.\frac{b_2}{a_1}\right|_{a_2=0} \qquad S_{12} = \left.\frac{b_1}{a_2}\right|_{a_1=0}$$

- S_{21} and S_{12} are signal amplitude gain when the other port is matched

Scattering matrix – S



$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \cdot \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$

$$|S_{21}|^2 = \frac{Power\ in\ Z_0\ load}{Power\ from\ Z_0\ source}$$

- a,b
  - information about signal power **AND** signal phase
- S_{ij}
  - network effect (gain) over signal power **including** phase information

Measuring S parameters - VNA

- Vector Network Analyzer

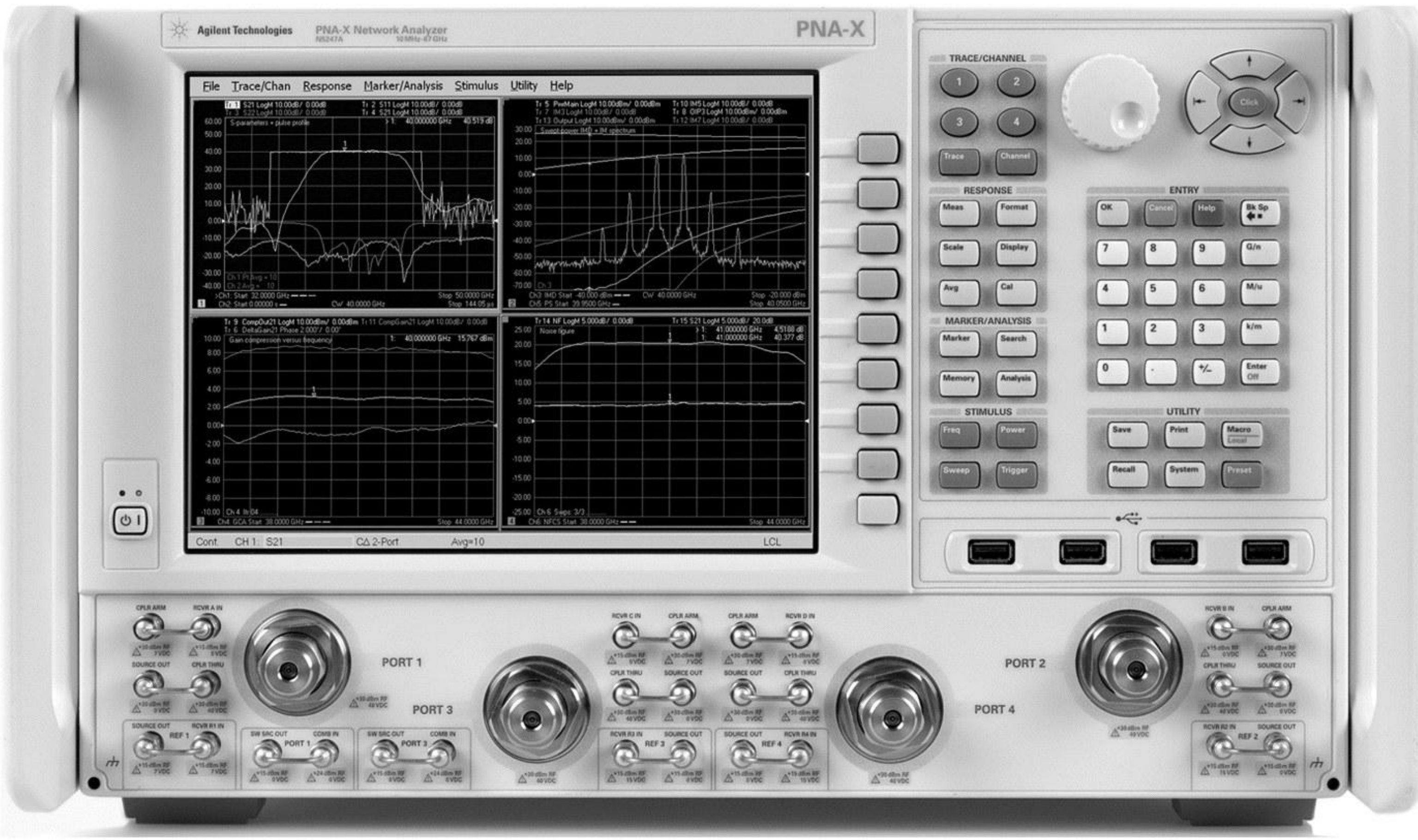


Figure 4.7
Courtesy of Agilent Technologies

Power dividers and directional couplers

Power dividers and couplers

- Desired functionality:
  - division
  - combining
- of signal power

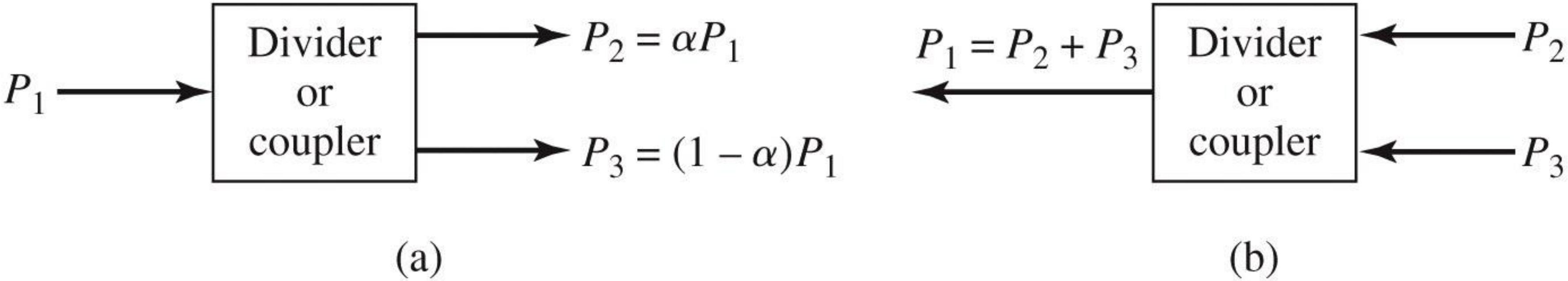


Figure 7.1

Three-Port Networks

$$[S]=\begin{bmatrix} 0 & S_{12} & S_{13} \\ S_{12} & 0 & S_{23} \\ S_{13} & S_{23} & 0 \end{bmatrix}$$

- 6 equations / 3 unknowns
  - no solution is possible
- A three-port network **cannot** be simultaneously:
  - reciprocal
  - lossless
  - matched at all ports
- If any one of these three conditions is relaxed, then a physically realizable device is possible

Four-Port Networks

- A four-port network simultaneously:
  - matched at all ports
  - reciprocal
  - lossless
- is **always** **directional**
  - the signal power injected into one port is transmitted **only towards two** of the other three ports

$$[S]=\begin{bmatrix} 0 & \alpha & \beta\cdot e^{j\theta} & 0 \\ \alpha & 0 & 0 & \beta\cdot e^{j\phi} \\ \beta\cdot e^{j\theta} & 0 & 0 & \alpha \\ 0 & \beta\cdot e^{j\phi} & \alpha & 0 \end{bmatrix}$$

Directional Couplers

Laboratory no. 2

Directional Coupler

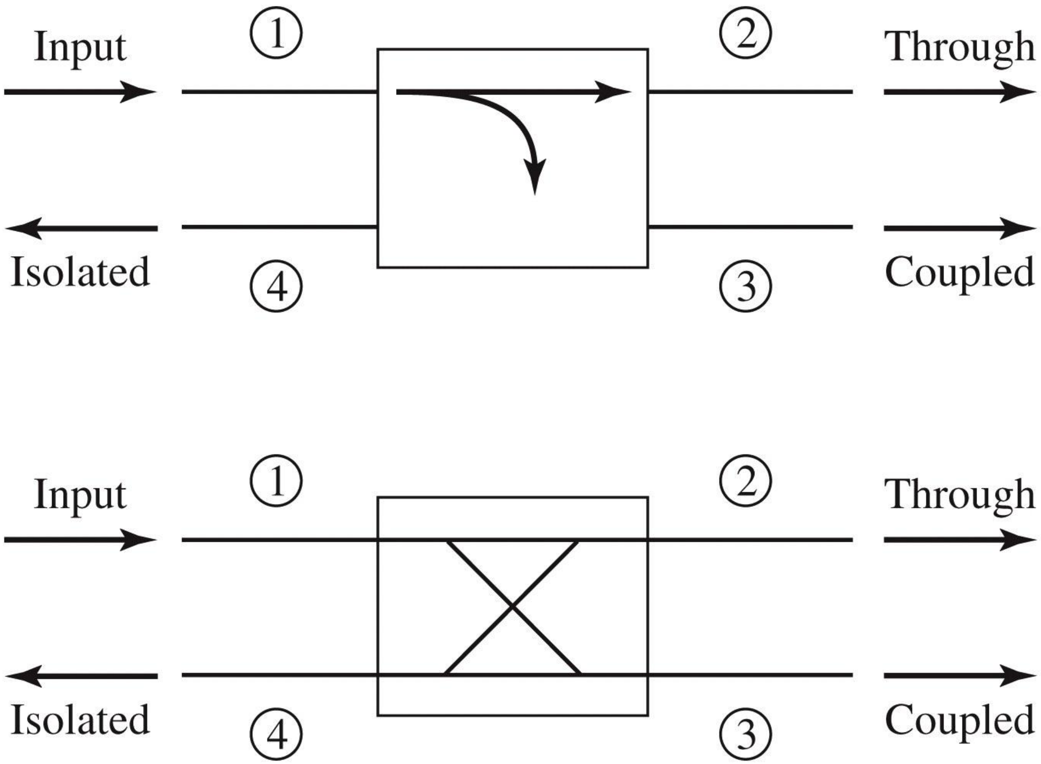


Figure 7.4

$$\left|S_{12}\right|^2 = \alpha^2 = 1 - \beta^2$$

$$\left|S_{13}\right|^2 = \beta^2$$

Cuplaj

$$C = 10\log\frac{P_1}{P_3} = -20\cdot\log(\beta)\,[\text{dB}]$$

Directivitate

$$D = 10\log\frac{P_3}{P_4} = 20\cdot\log\left(\frac{\beta}{\left|S_{14}\right|}\right)[\text{dB}]$$

Izolare

$$I = 10\log\frac{P_1}{P_4} = -20\cdot\log\left|S_{14}\right|\,[\text{dB}]$$

$$I = D + C\,,\ \ dB$$

Quadrature coupler

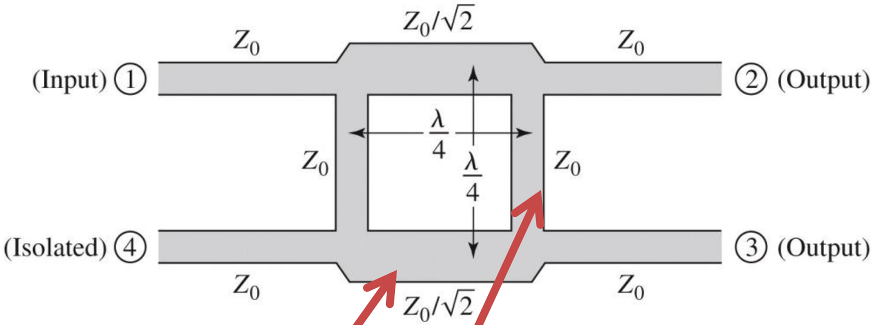


Figure 7.21

$$y_2^2 = 1 + y_1^2$$

$$|\beta| = \frac{\sqrt{y_2^2 - 1}}{y_2}$$

$$C[\text{dB}] = -20 \cdot \log_{10} \frac{\sqrt{y_2^2 - 1}}{y_2}$$

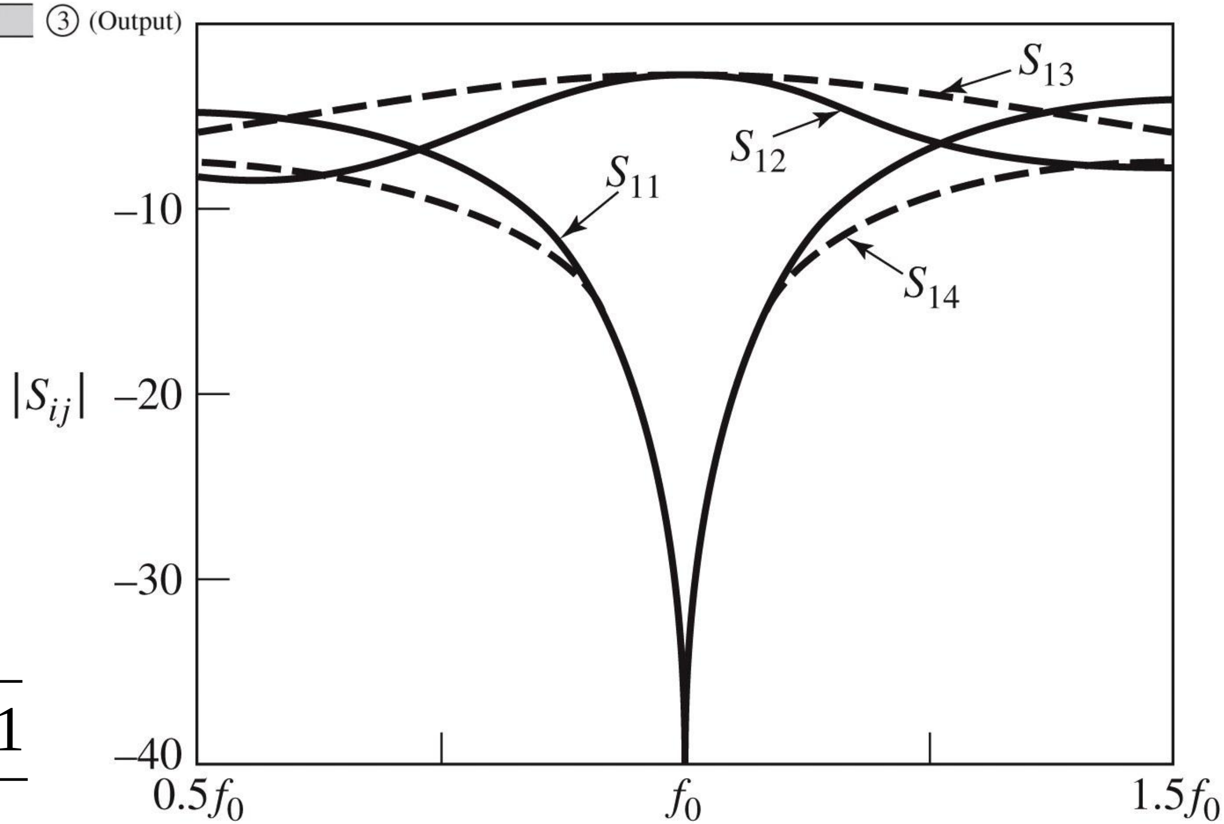
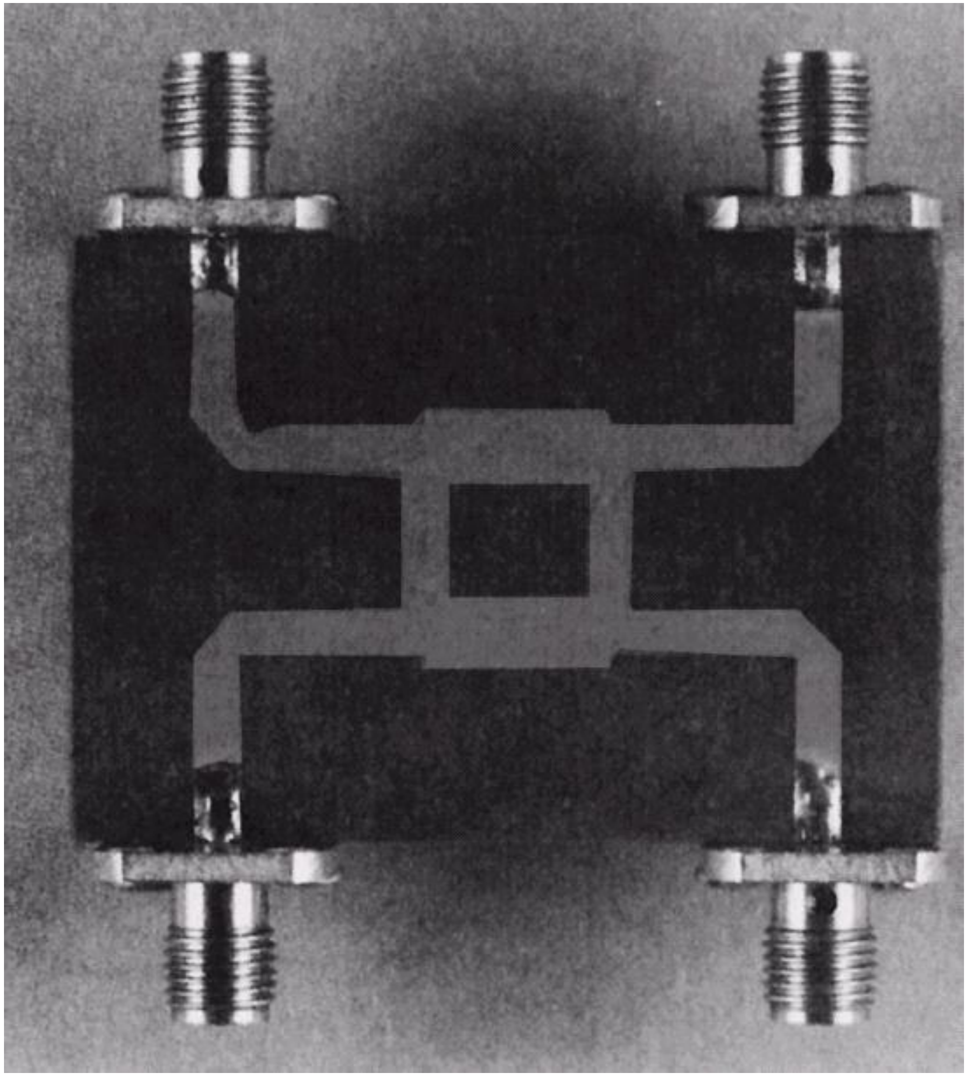
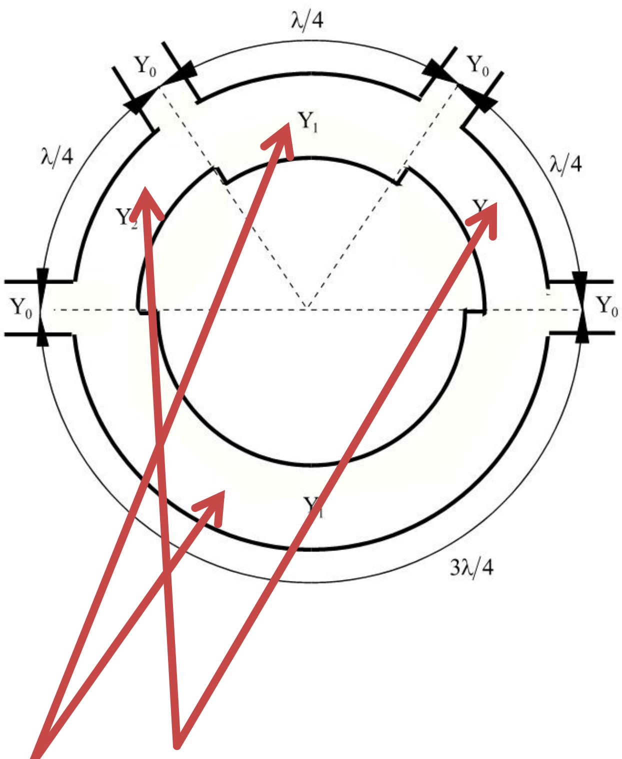


Figure 7.25

Quadrature coupler



Ring coupler



$$y_1^2 + y_2^2 = 1$$

$$|\beta| = y_1$$

$$C\,[\text{dB}] = -20 \cdot \log_{10}(y_1)$$

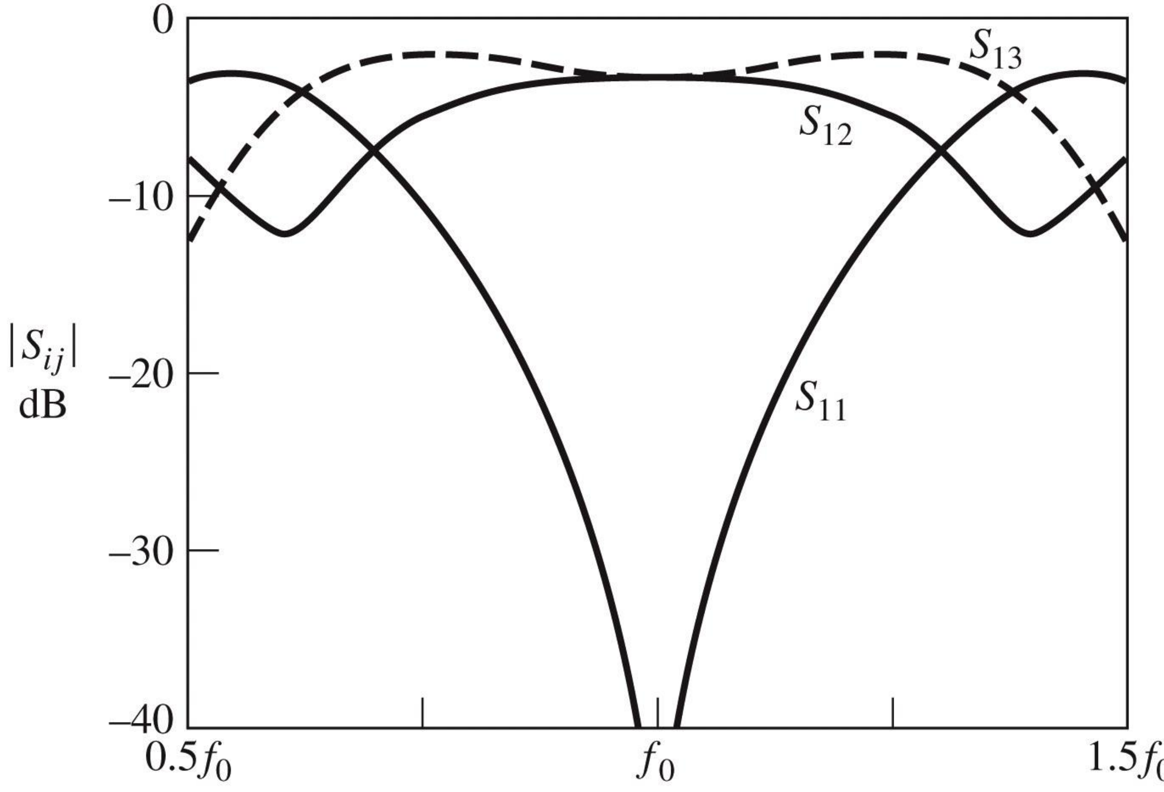


Figure 7.46

Ring coupler

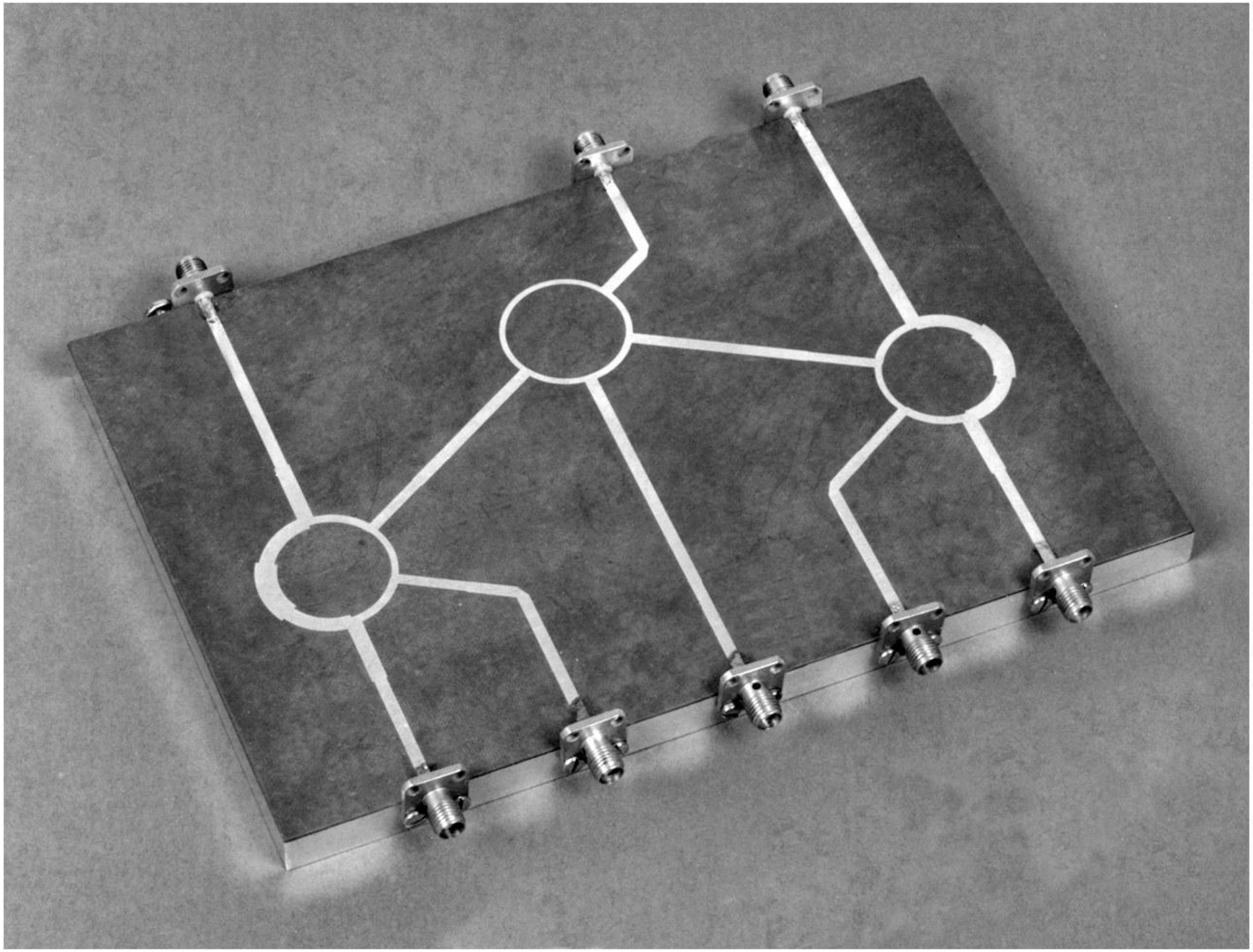
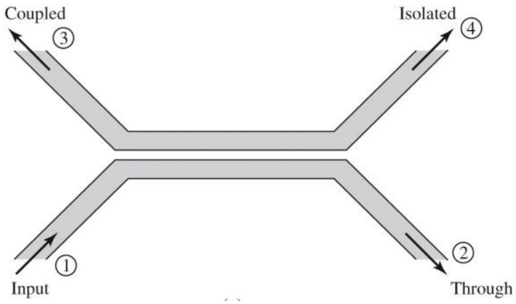


Figure 7.43
Courtesy of M. D. Abouzahra, MIT Lincoln Laboratory, Lexington, Mass.

Coupled lines coupler



$$Z_{ce} Z_{co} = Z_0^2$$

$$|\beta| = \frac{Z_{ce} - Z_{co}}{Z_{ce} + Z_{co}}$$

$$C\,[\mathrm{dB}] = -20 \cdot \log_{10}\left(\frac{Z_{ce} - Z_{co}}{Z_{ce} + Z_{co}}\right)$$

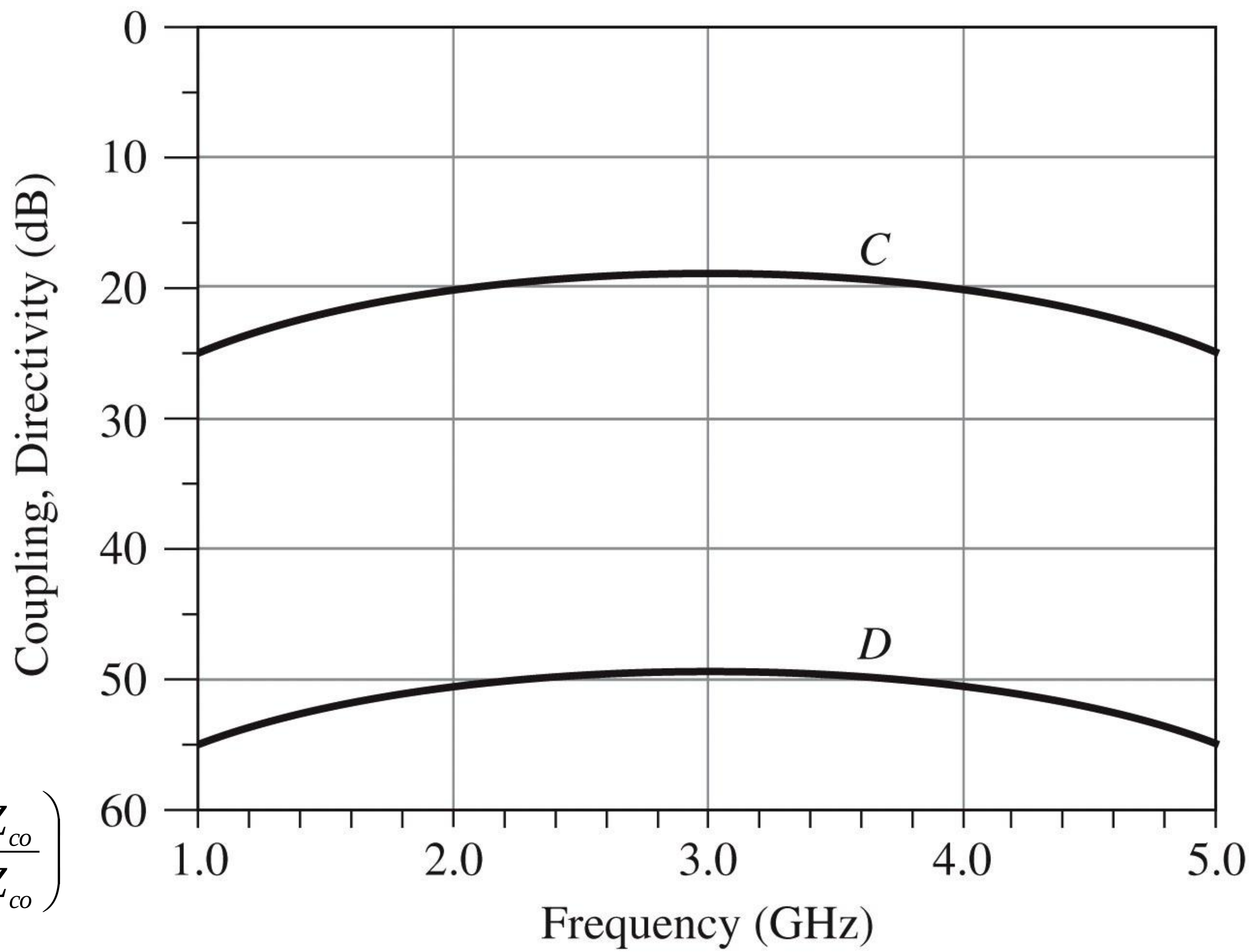
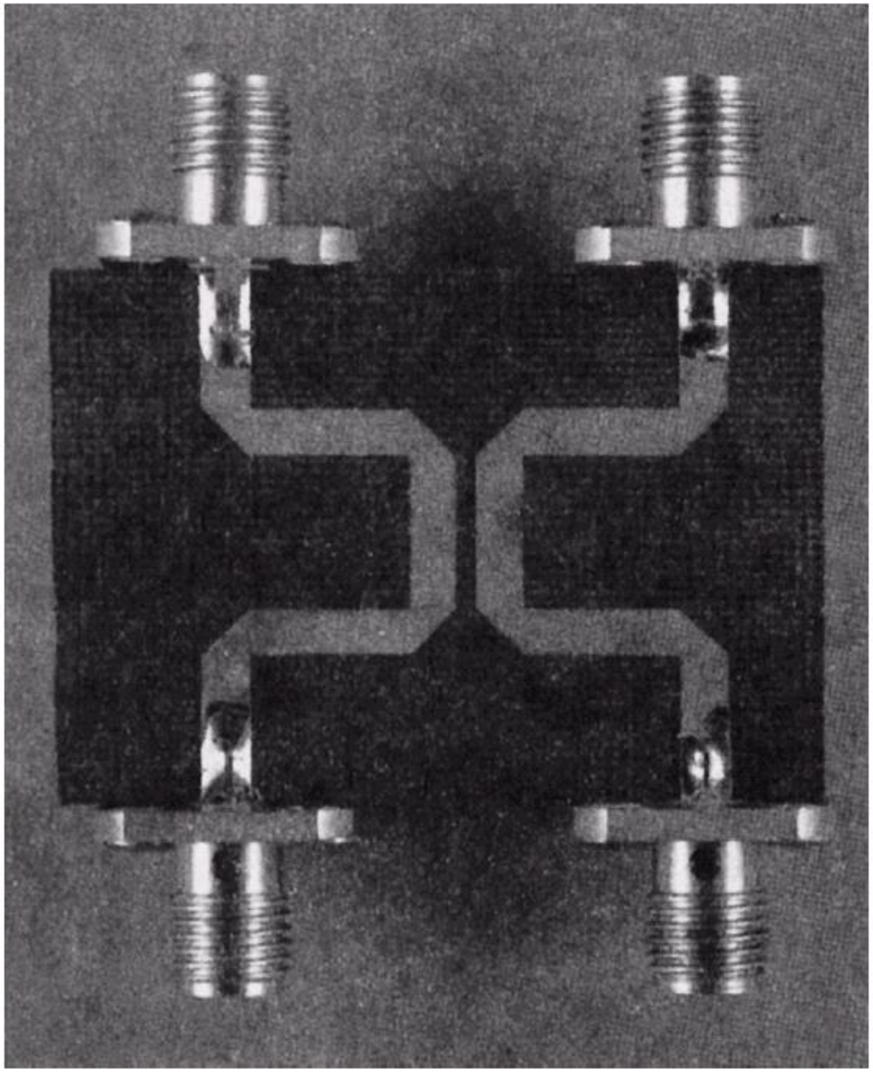


Figure 7.34

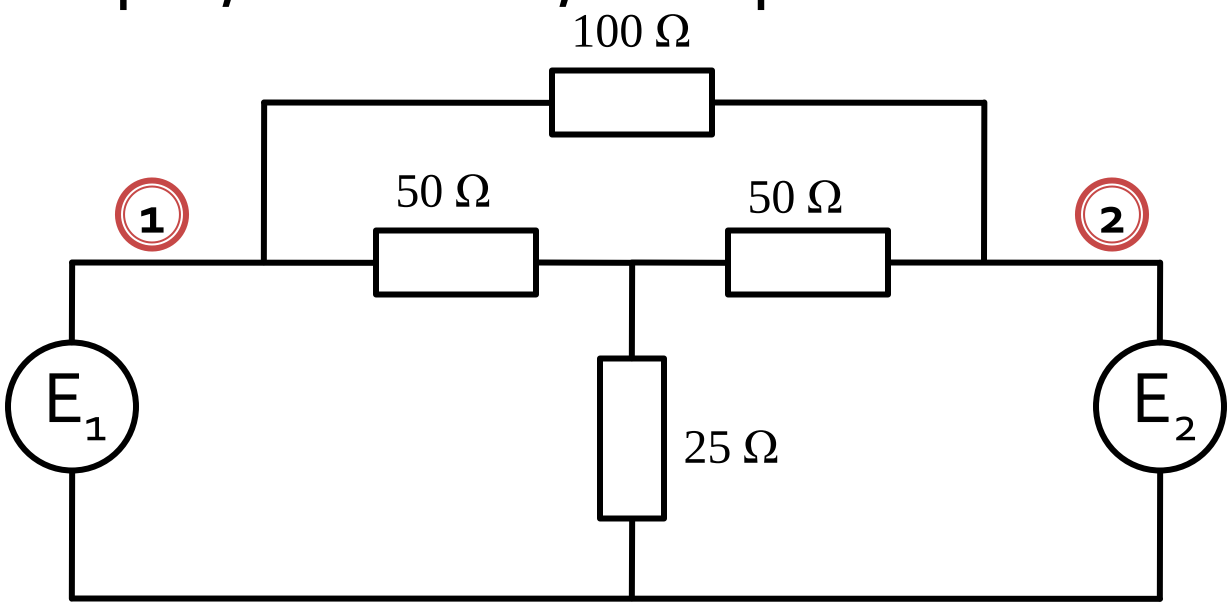
Coupled lines coupler



Microwave Network Analysis

Even/Odd Mode Analysis

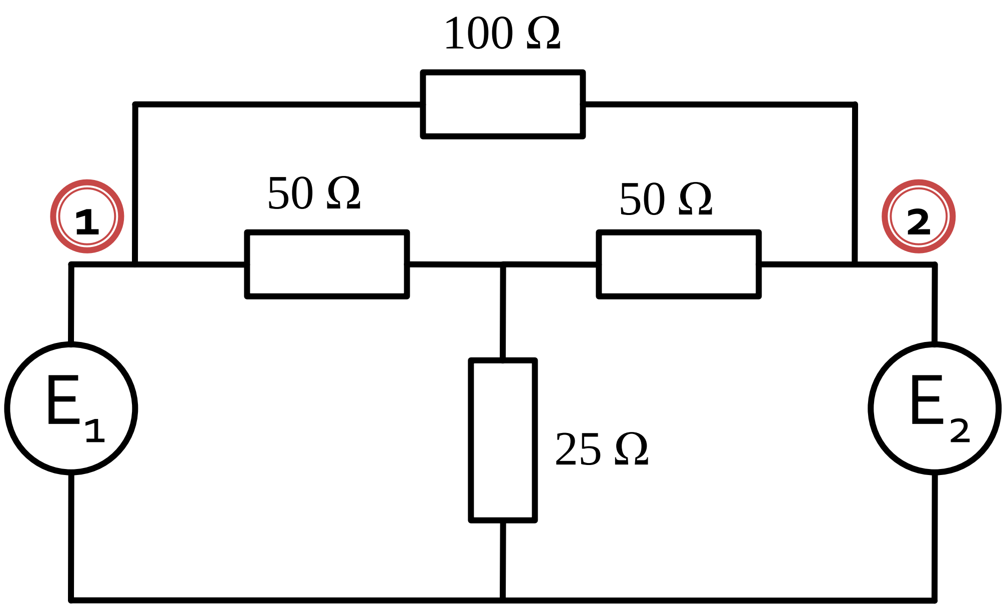
- useful method, necessary even for multiple ports
- example, resistors, two port circuitcircuit

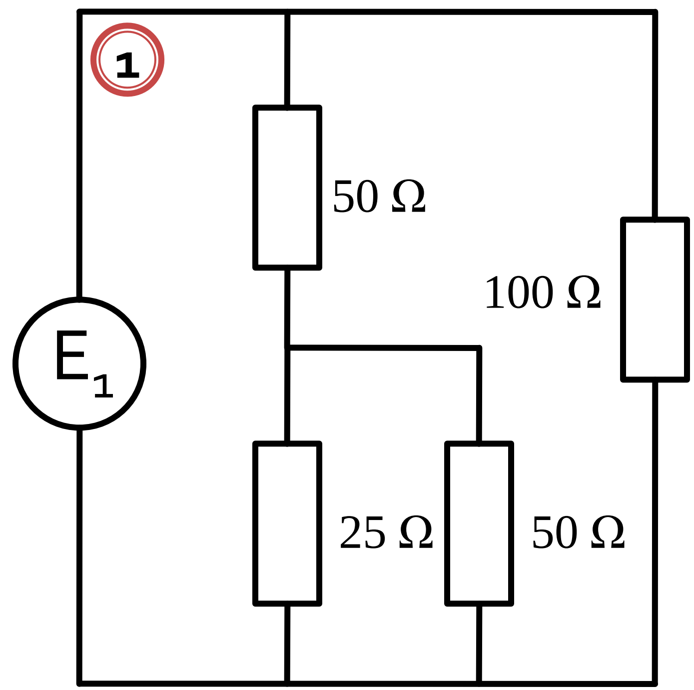


Even/Odd Mode Analysis

- assume we want to compute Y_{11}
- $E_2 = 0$

$$Y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0}$$



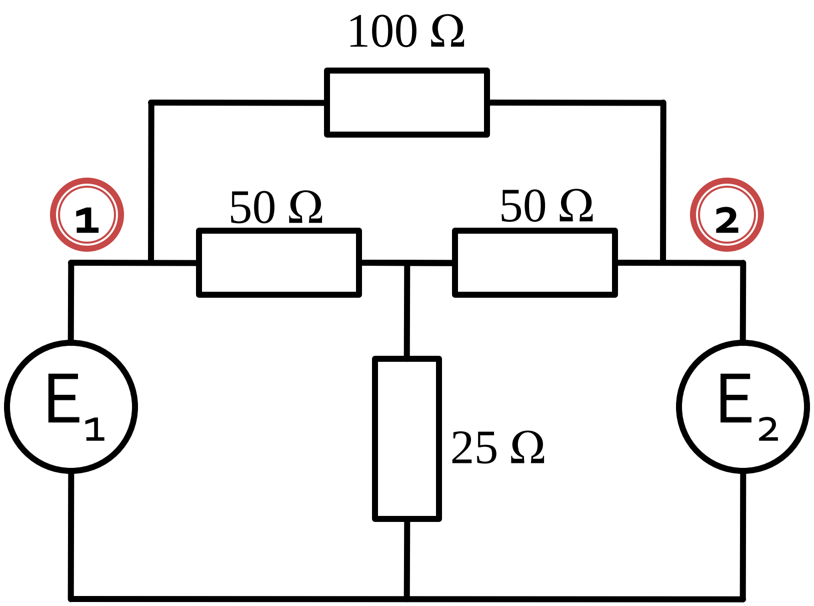


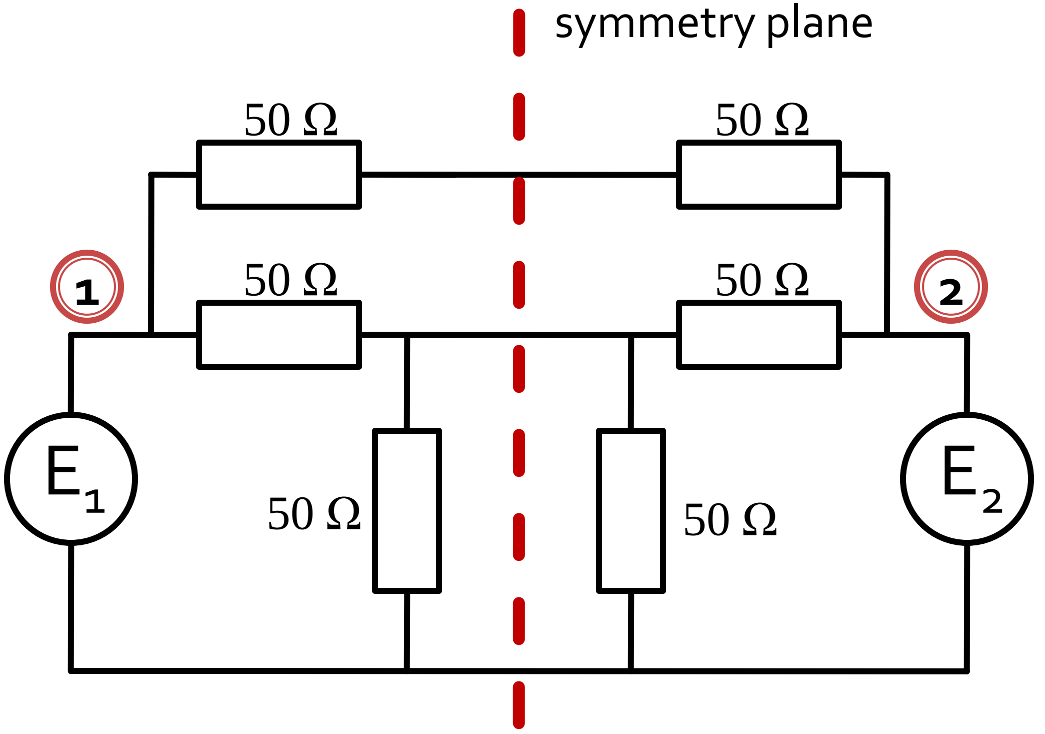
$$R_{ech} = 100\Omega \,||\, (50\Omega + 25\Omega \,||\, 50\Omega) =$$
$$= 100\Omega \,||\, (50\Omega + 16.67\Omega) = 100\Omega \,||\, 66.67\Omega = 40\Omega$$

$$Y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} = 0.025S$$

Even/Odd Mode Analysis

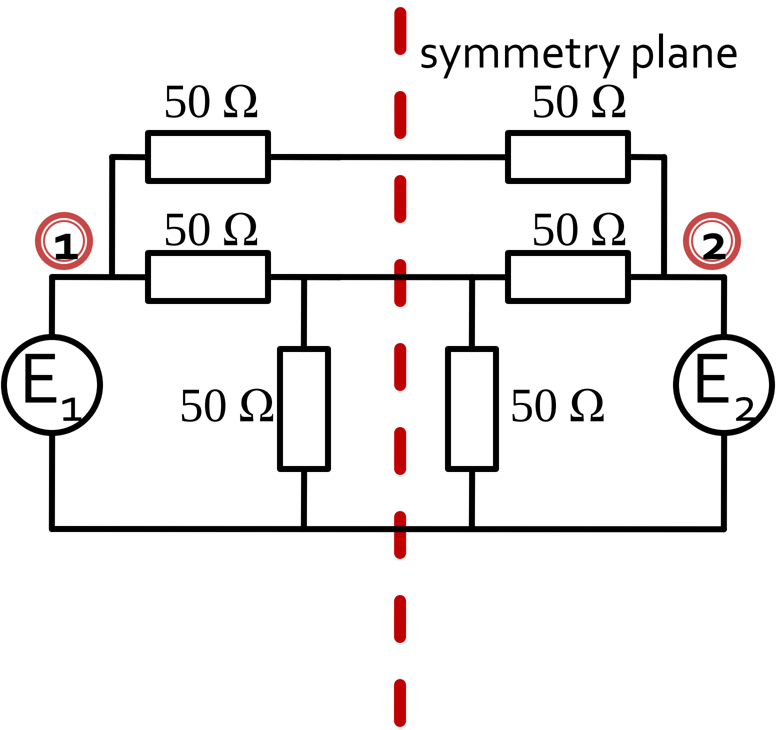
- Even/Odd mode analysis benefit from the existence of symmetry planes in the circuit
  - existing or
  - created (forced)

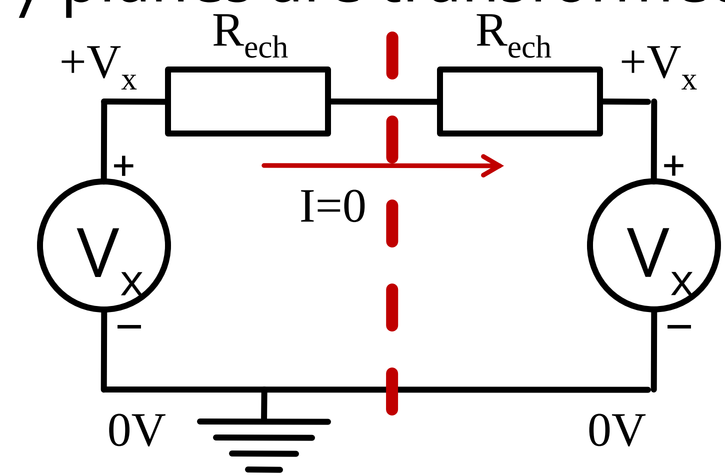




Even/Odd Mode Analysis

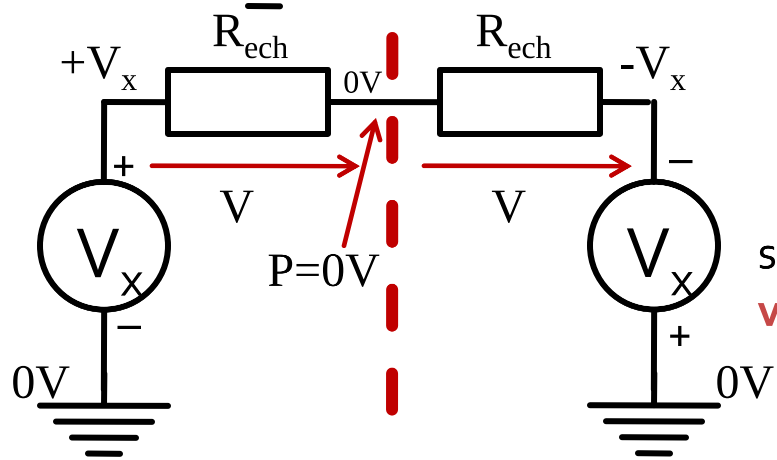
- when exciting the ports with symmetric/anti-symmetric sources the symmetry planes are transformed into:
  - open circuit
  - virtual ground





$$I = 0, \forall V_x$$

symmetry plane
open circuit

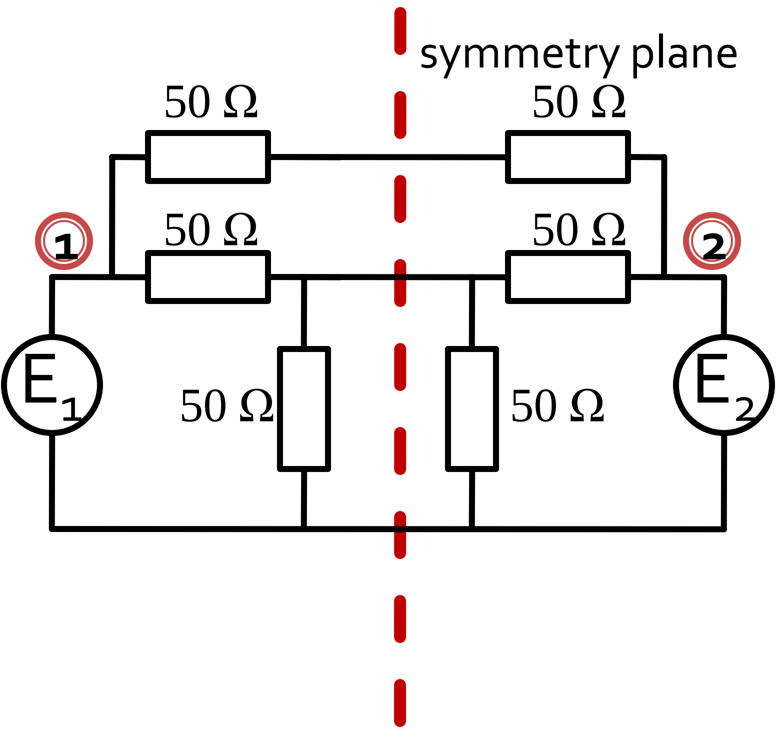


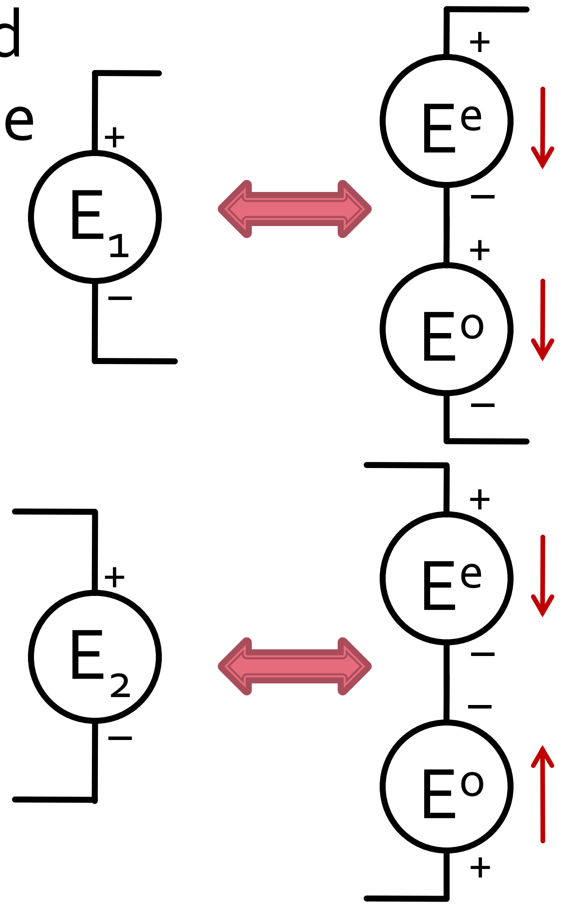
$$P = 0, \forall V_x$$

symmetry plane
virtual ground

Even/Odd Mode Analysis

- the combination of any two sources is equivalent for linear circuits with the superposition of:
  - a symmetric source and
  - a anti-symmetric source





$$E_1 = E^e + E^o$$

$$E_2 = E^e - E^o$$

$$E^e = \frac{E_1 + E_2}{2}$$

$$E^o = \frac{E_1 - E_2}{2}$$

Even/Odd Mode Analysis

- In linear circuits the **superposition principle** is always true
  - the response caused by two or more stimuli is the sum of the responses that would have been caused by each stimulus individually

Response (Source1 + Source2) =

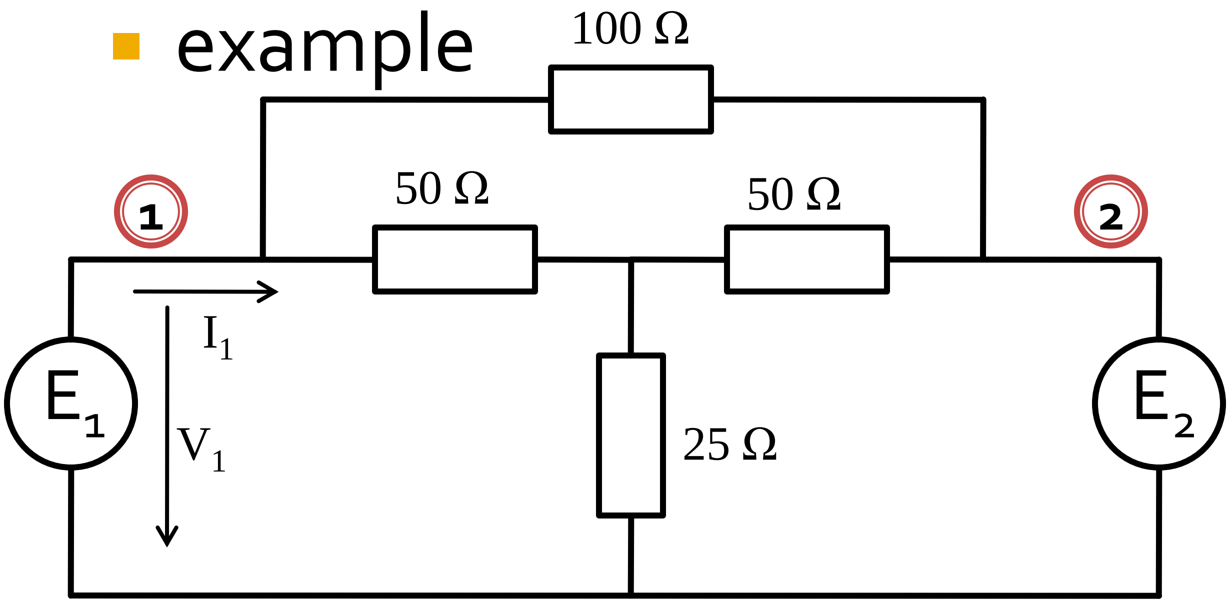
= Response (Source1) + Response (Source2)

Response(ODD + EVEN) = Response (ODD) + Response (EVEN)

We can benefit from existing symmetries !!

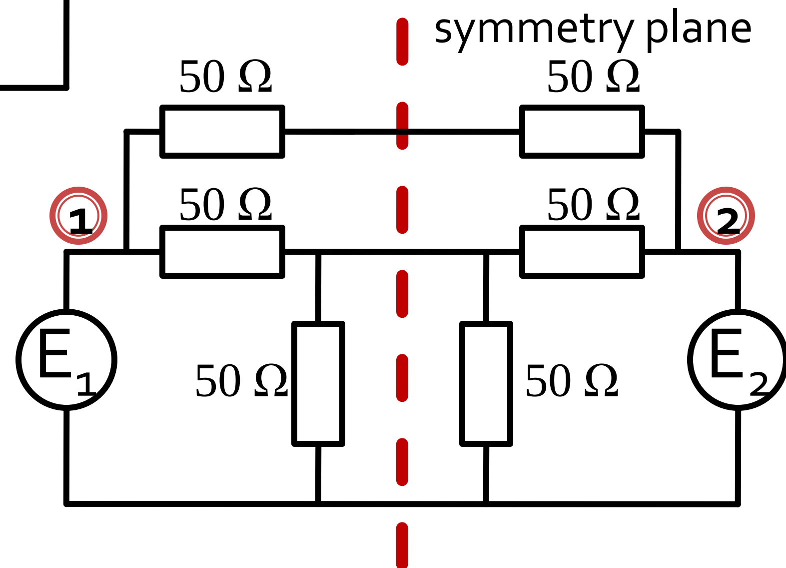
Even/Odd Mode Analysis

- example



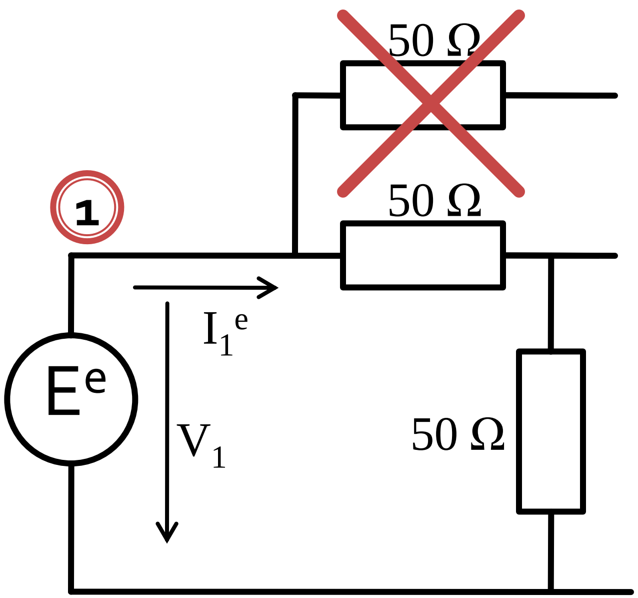
$$Y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0}$$

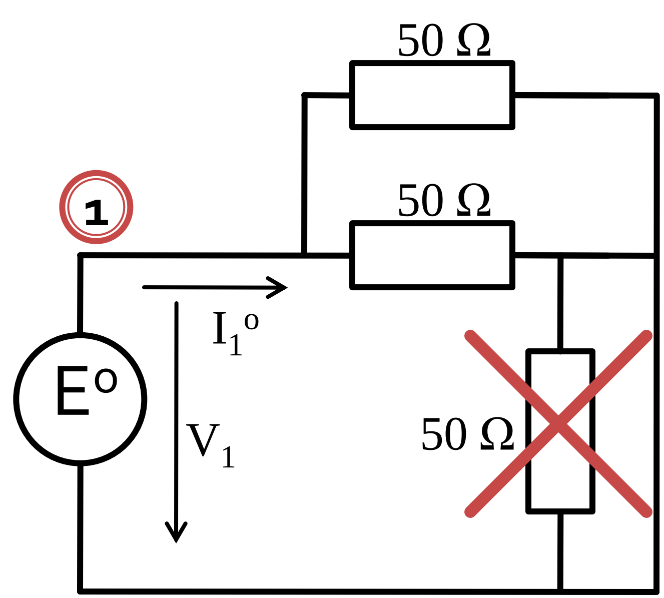
$$V_2 \equiv E_2 = 0 \Rightarrow \quad E^e = \frac{E_1}{2} \qquad E^o = \frac{E_1}{2}$$



Even/Odd Mode Analysis

- Even/Odd mode analysis





$$R_{ech}^{e} = 50\Omega + 50\Omega = 100\Omega$$

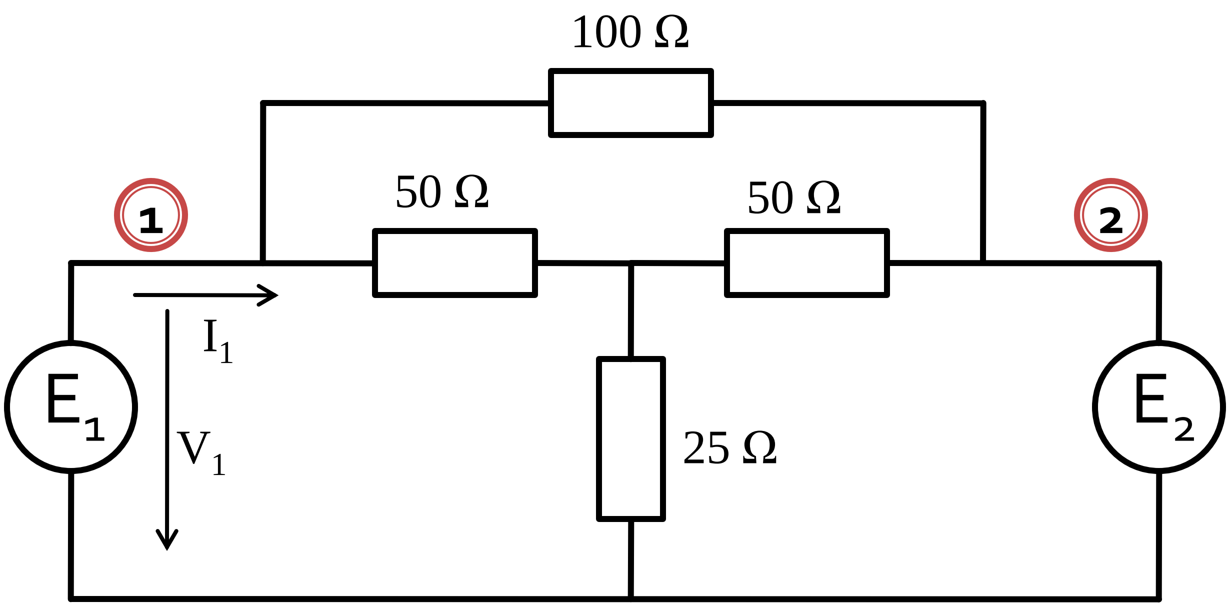
$$I_1^e = \frac{E^e}{R_{ech}^e} = \frac{E_1/2}{100\Omega} = \frac{E_1}{200\Omega}$$

$$R_{ech}^{o} = 50\Omega \,||\, 50\Omega = 25\Omega$$

$$I_1^o = \frac{E^o}{R_{ech}^o} = \frac{E_1/2}{25\Omega} = \frac{E_1}{50\Omega}$$

Even/Odd Mode Analysis

- superposition principle



$$I_1 = I_1^e + I_1^o$$

$$V_1 = V_1^e + V_1^o$$

$$I_1 = I_1^e + I_1^o = \frac{E_1}{200\Omega} + \frac{E_1}{50\Omega} = \frac{E_1}{40\Omega}$$

$$V_1 = V_1^e + V_1^o = E_1$$

$$Y_{11} = \frac{I_1}{V_1} = \frac{1}{40\Omega} = 0.025S$$

Even/Odd Mode Analysis

- In linear circuits we can use the superposition principle
- advantages
  - reduction of the circuit complexity
  - decrease in the number of ports (**main** advantage)

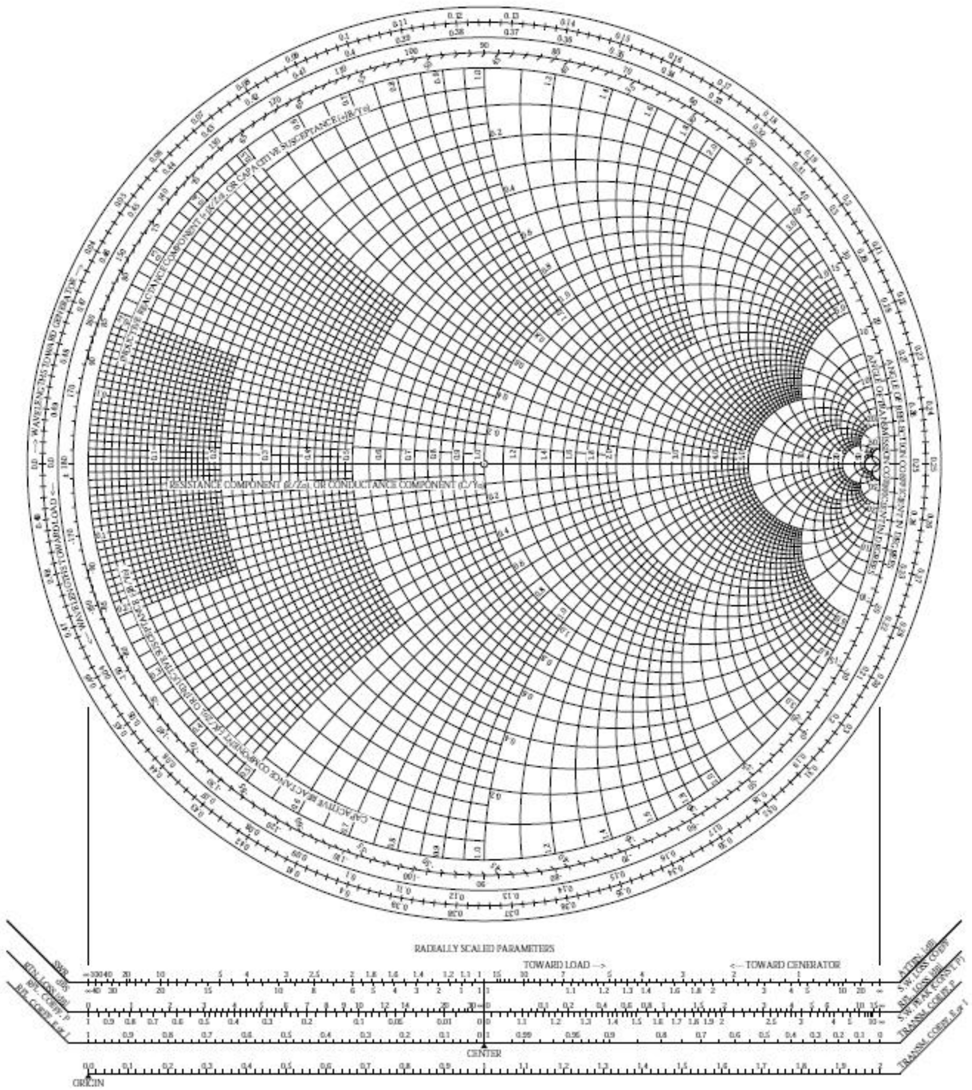
Response(ODD + EVEN) = Response (ODD) + Response (EVEN)

We can benefit from existing symmetries !!

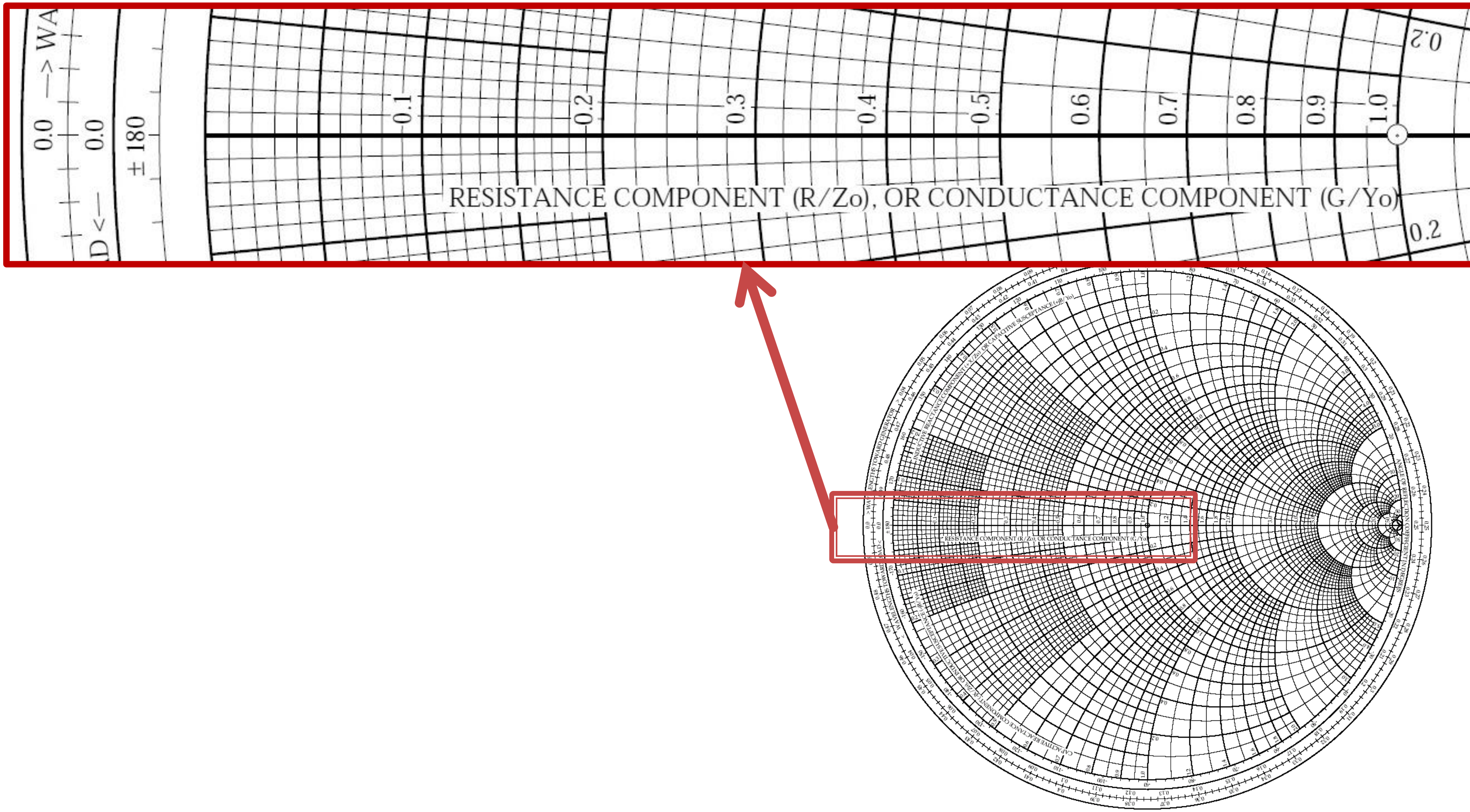
Impedance Matching

The Smith Chart

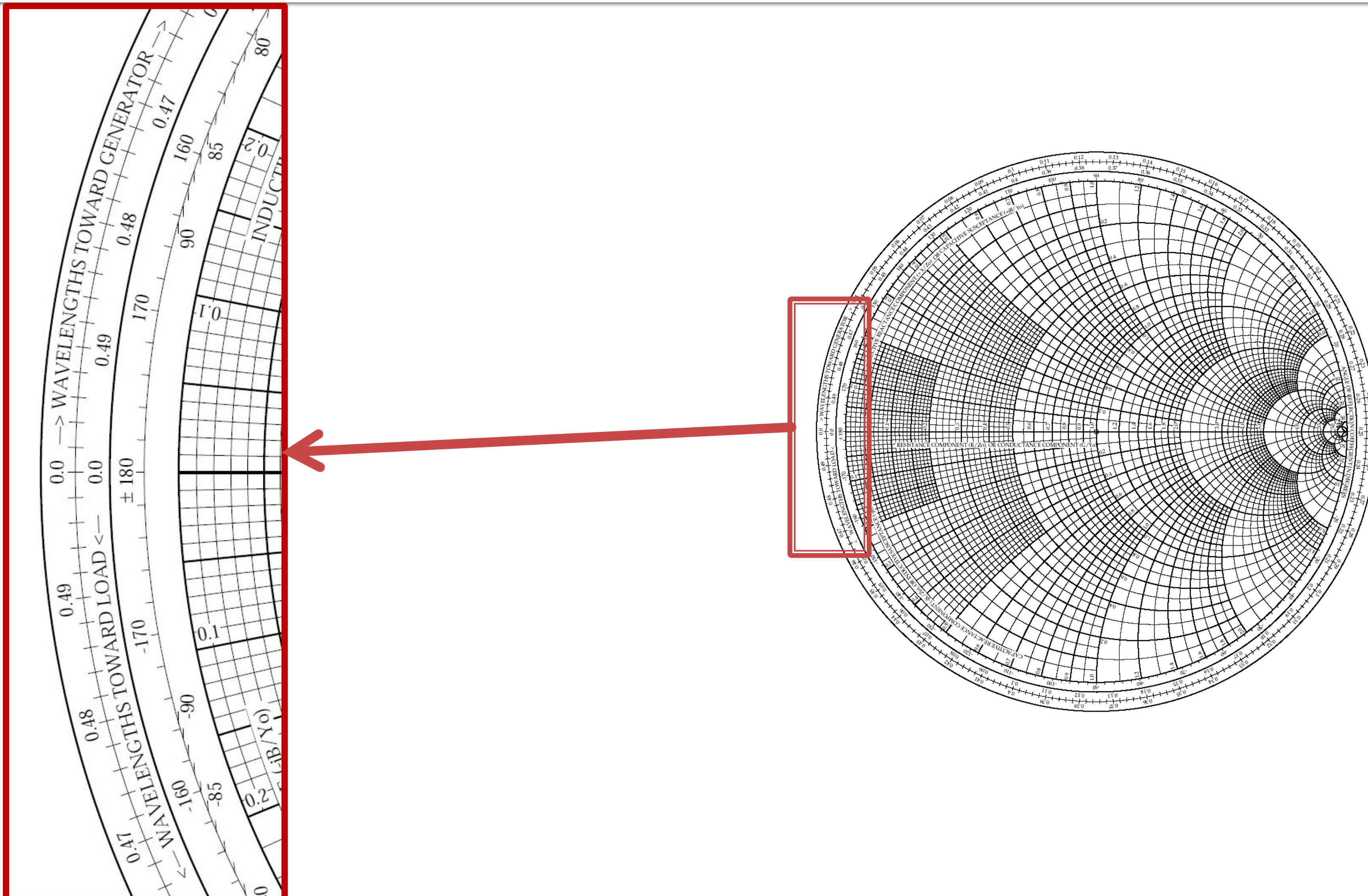
The Smith Chart



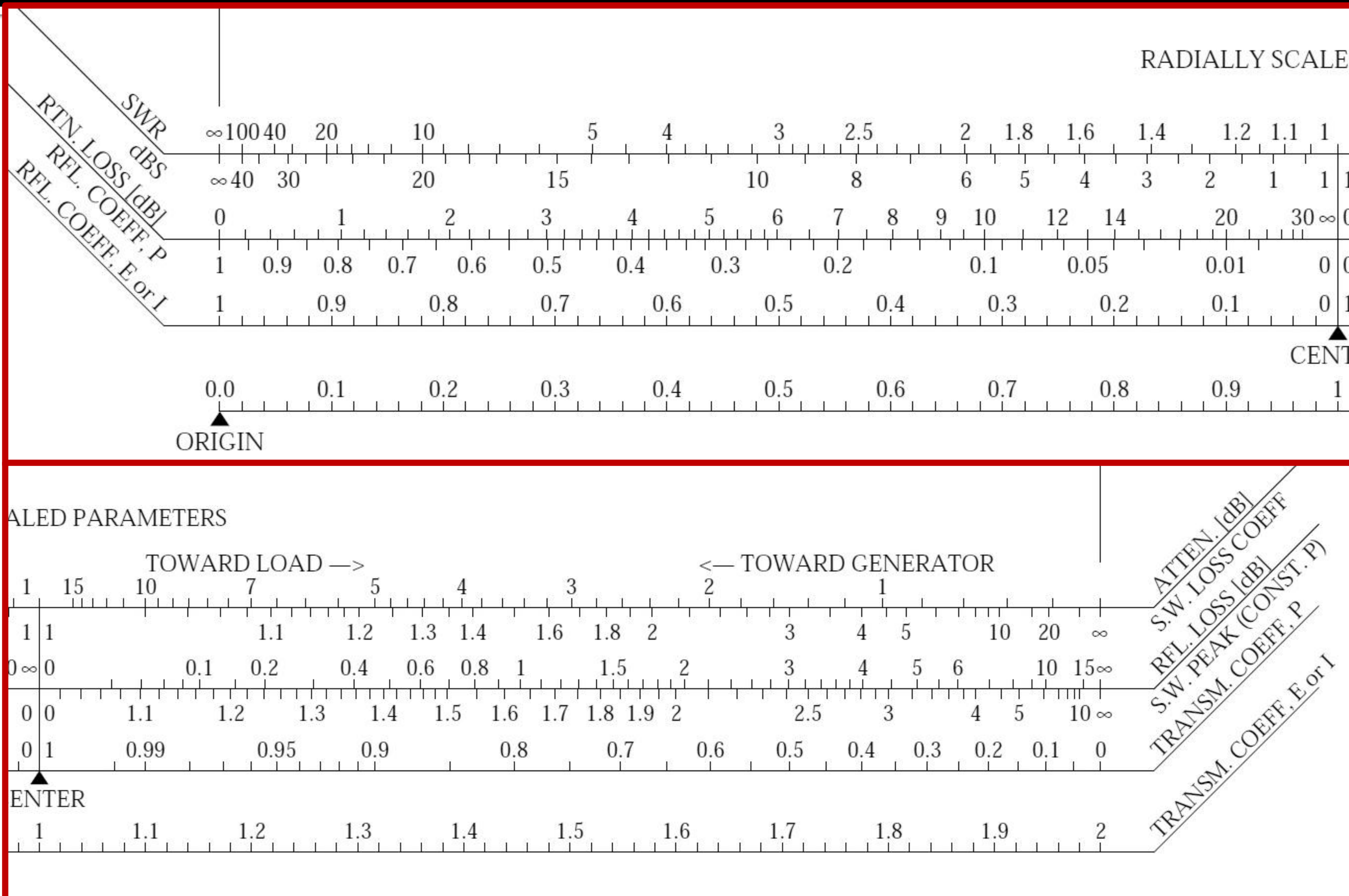
The Smith Chart



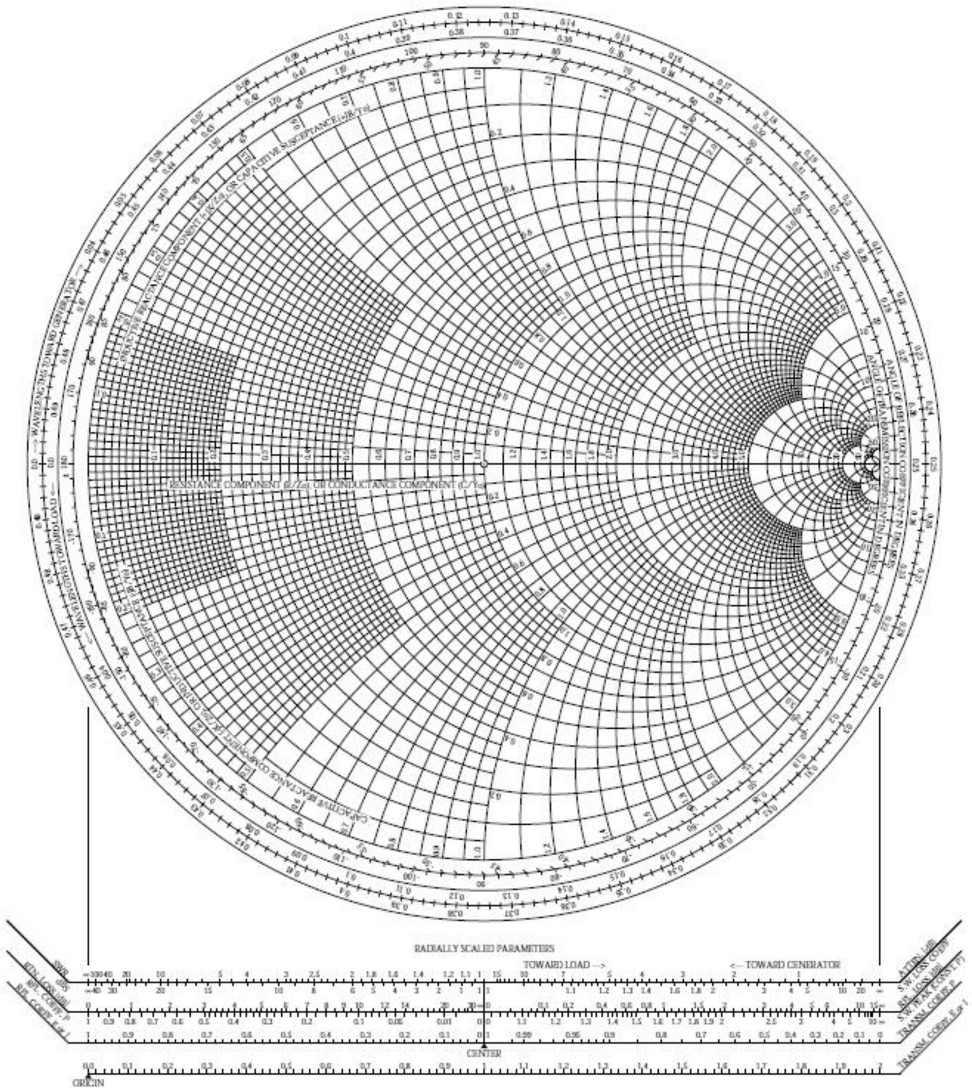
The Smith Chart



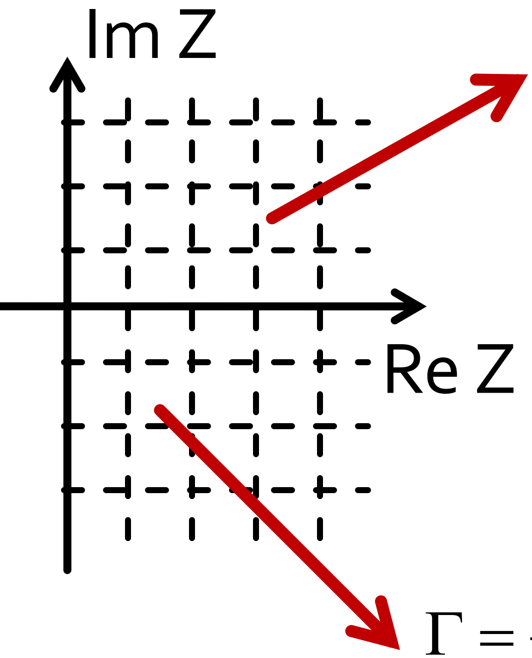
The Smith Chart



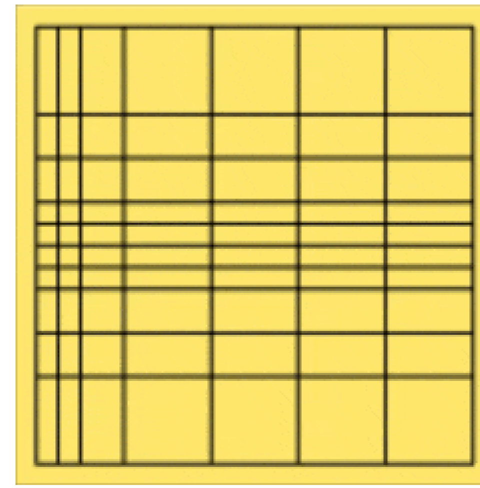
The Smith Chart



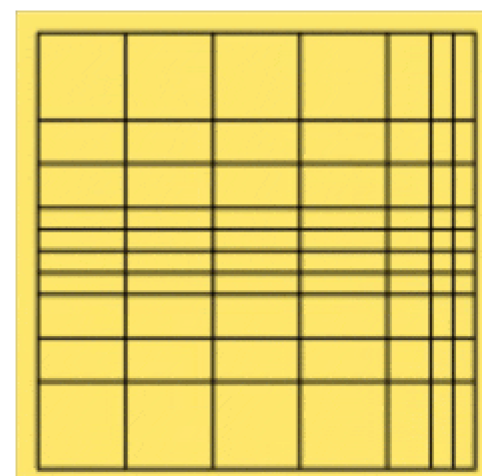
The Smith Chart



$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{z_L - 1}{z_L + 1}$$

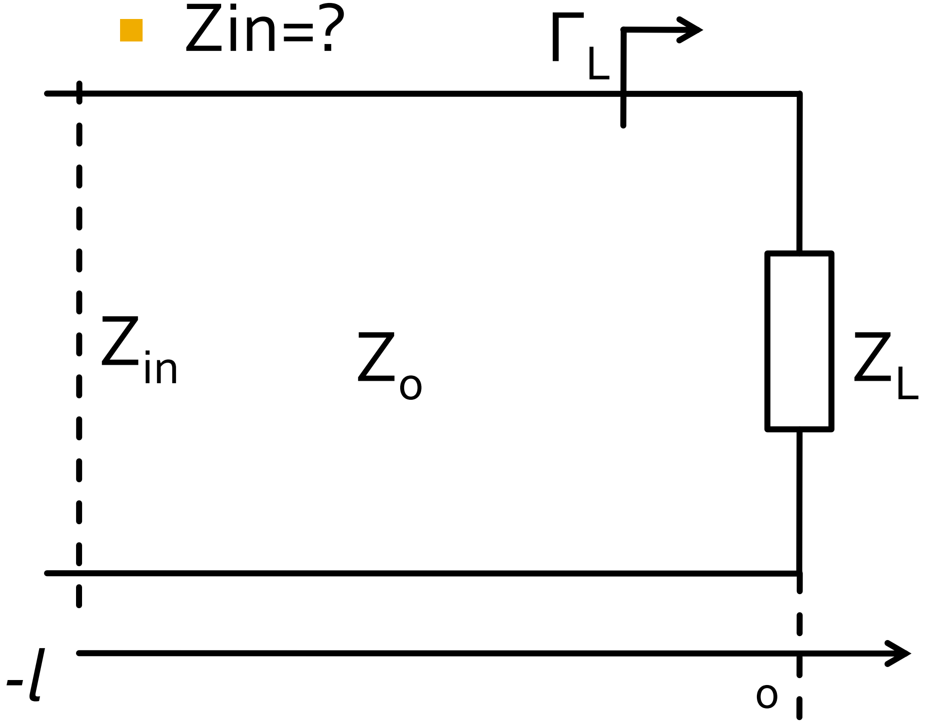


$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{Y_0 - Y_L}{Y_0 + Y_L} = \frac{1 - y_L}{1 + y_L}$$



Traditional usage

- transmission line
  - 100Ω characteristic impedance
  - 0.3λ length
  - Z_L = 40Ω+j·70Ω load
- Zin=?



$$Z_{in} = Z_0 \cdot \frac{Z_L + j \cdot Z_0 \cdot \tan \beta \cdot l}{Z_0 + j \cdot Z_L \cdot \tan \beta \cdot l}$$

$$Z_{in} = 36.5340\Omega - j \cdot 61.1190\Omega$$

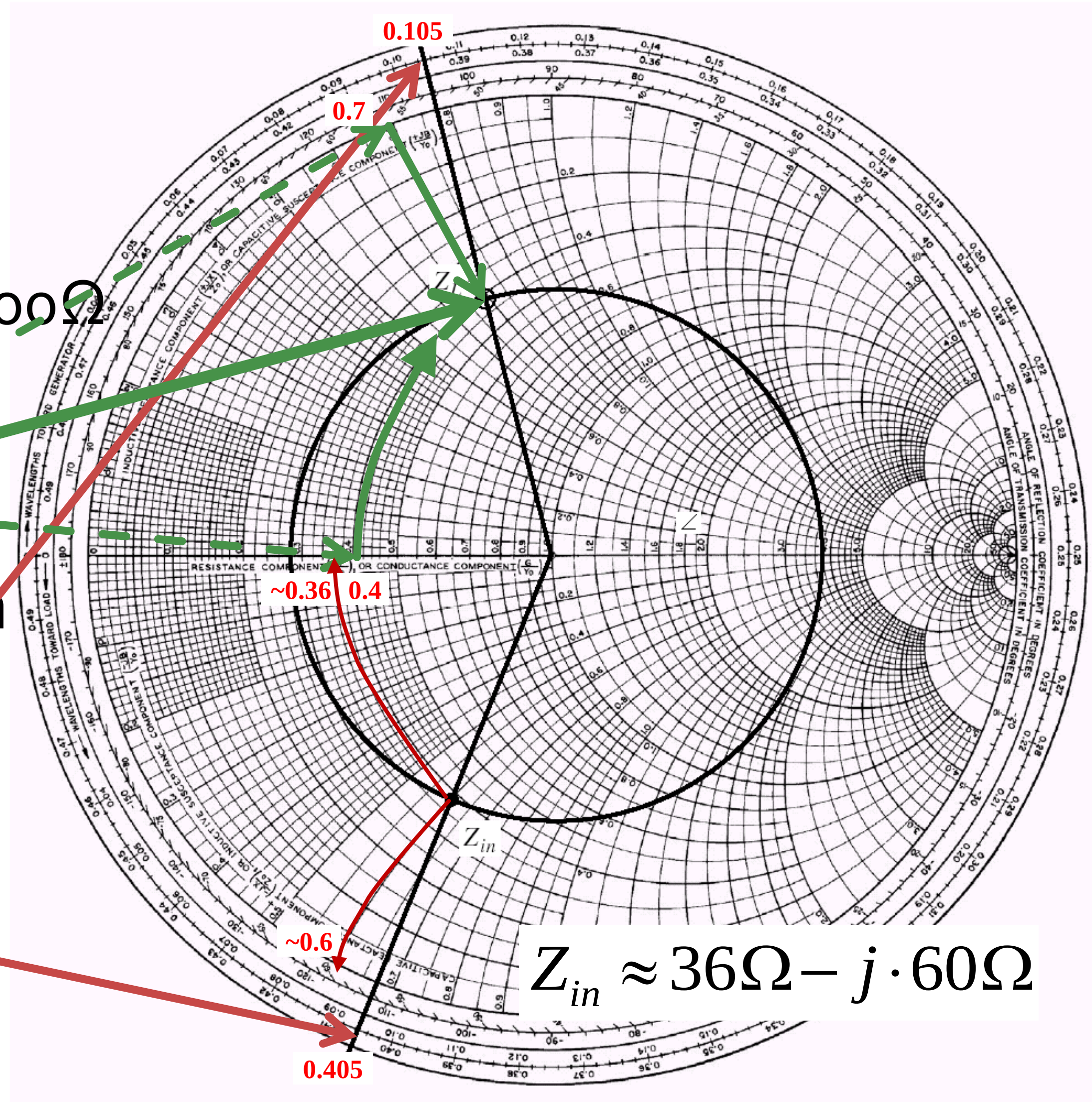
Traditional usage

- transmission line
  - 100Ω impedance
  - 0.3λ length
  - Z_L = 40Ω+j·70Ω load
- normalization with Z_0 = 100Ω

$$z_L = \frac{Z_L}{Z_0} = 0.4 + j \cdot 0.7$$

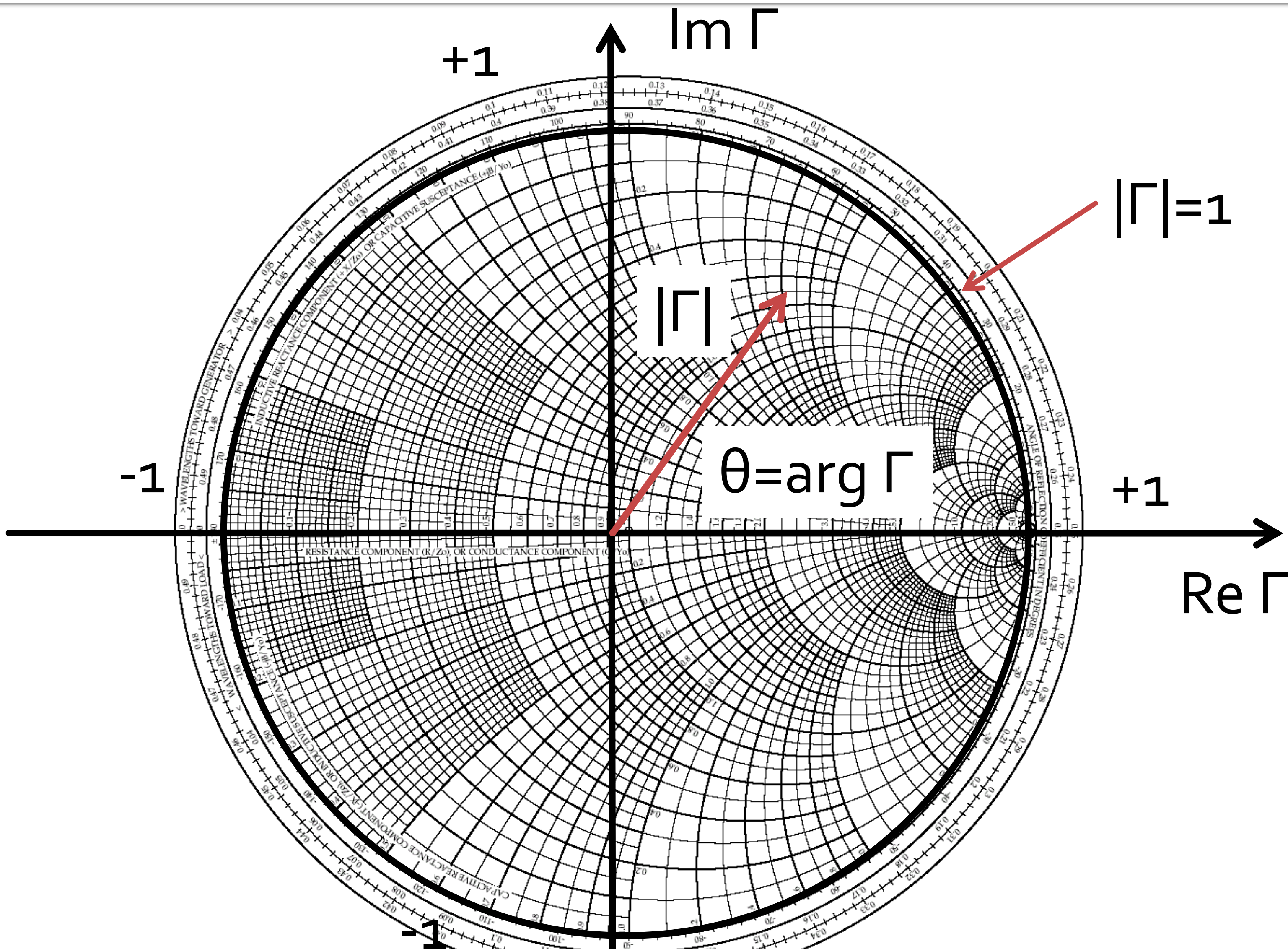
- movement with 0.3λ on a line with Z_0 = 100Ω (circle)
  - from z_L (0.105λ)
  - to z_{in} (0.405λ)

$$z_{in} \approx 0.36 - j \cdot 0.6 = \frac{Z_{in}}{Z_0}$$

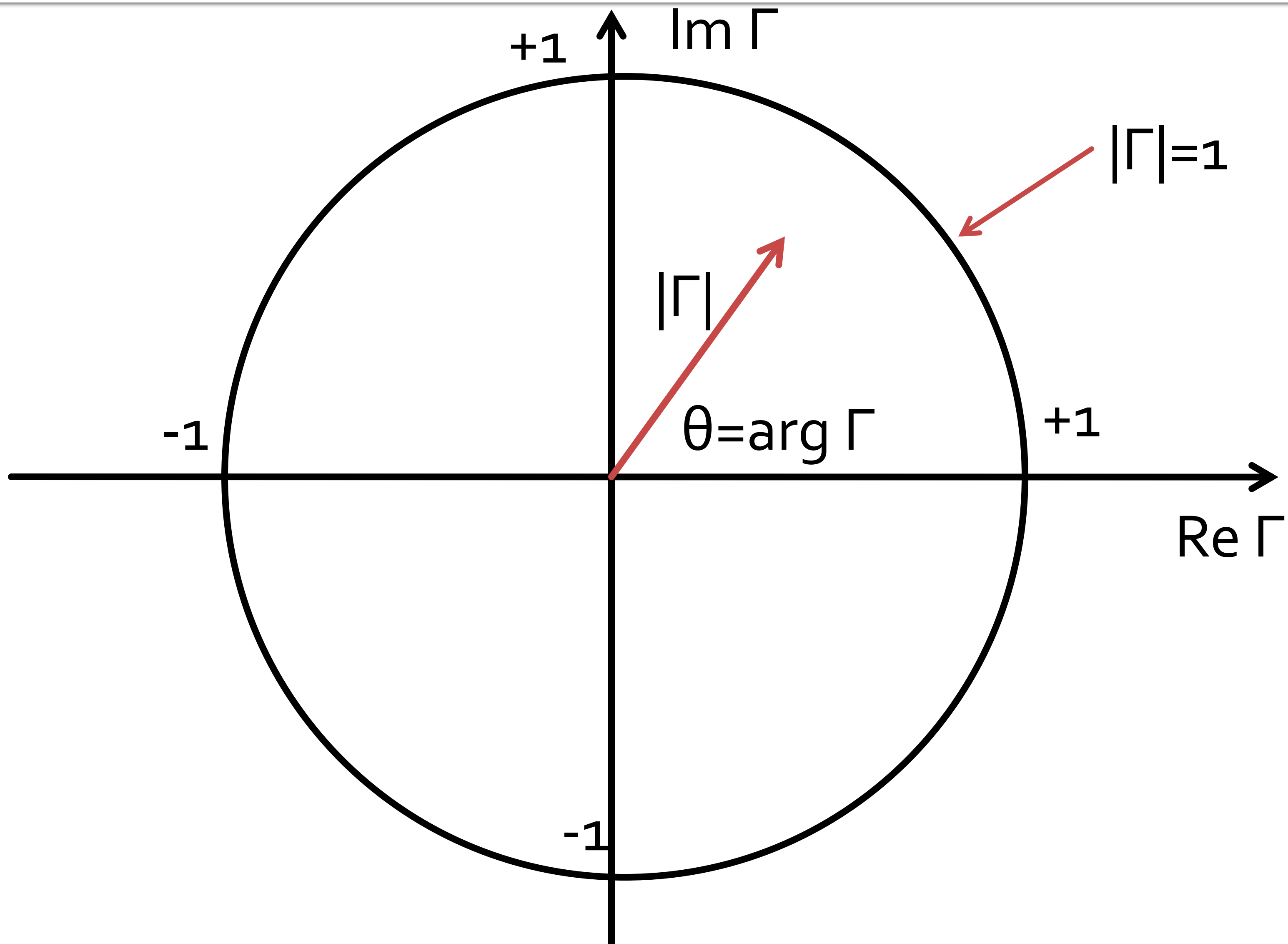


$$Z_{in} \approx 36\Omega - j \cdot 60\Omega$$

The Smith Chart

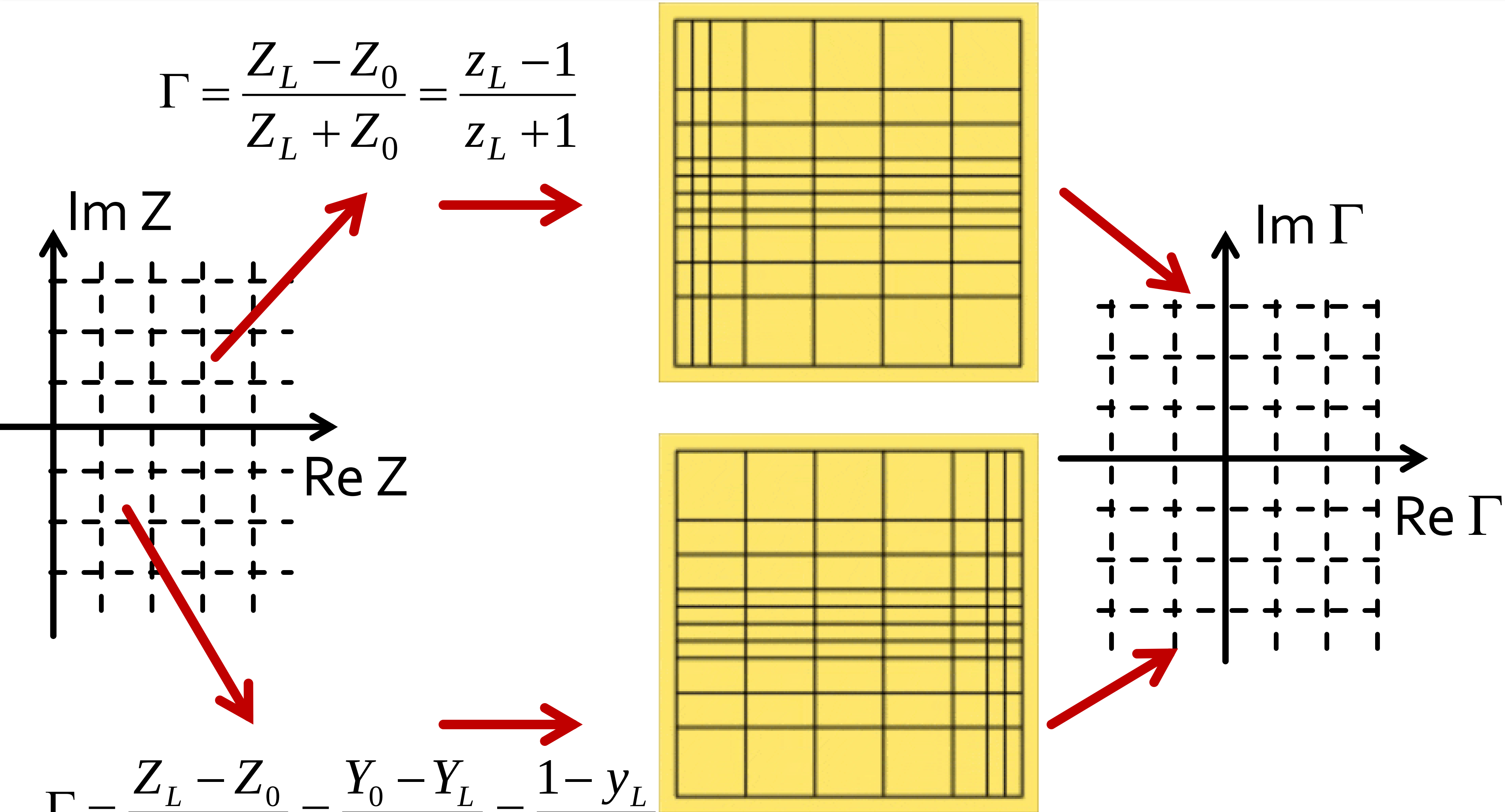


The Smith Chart



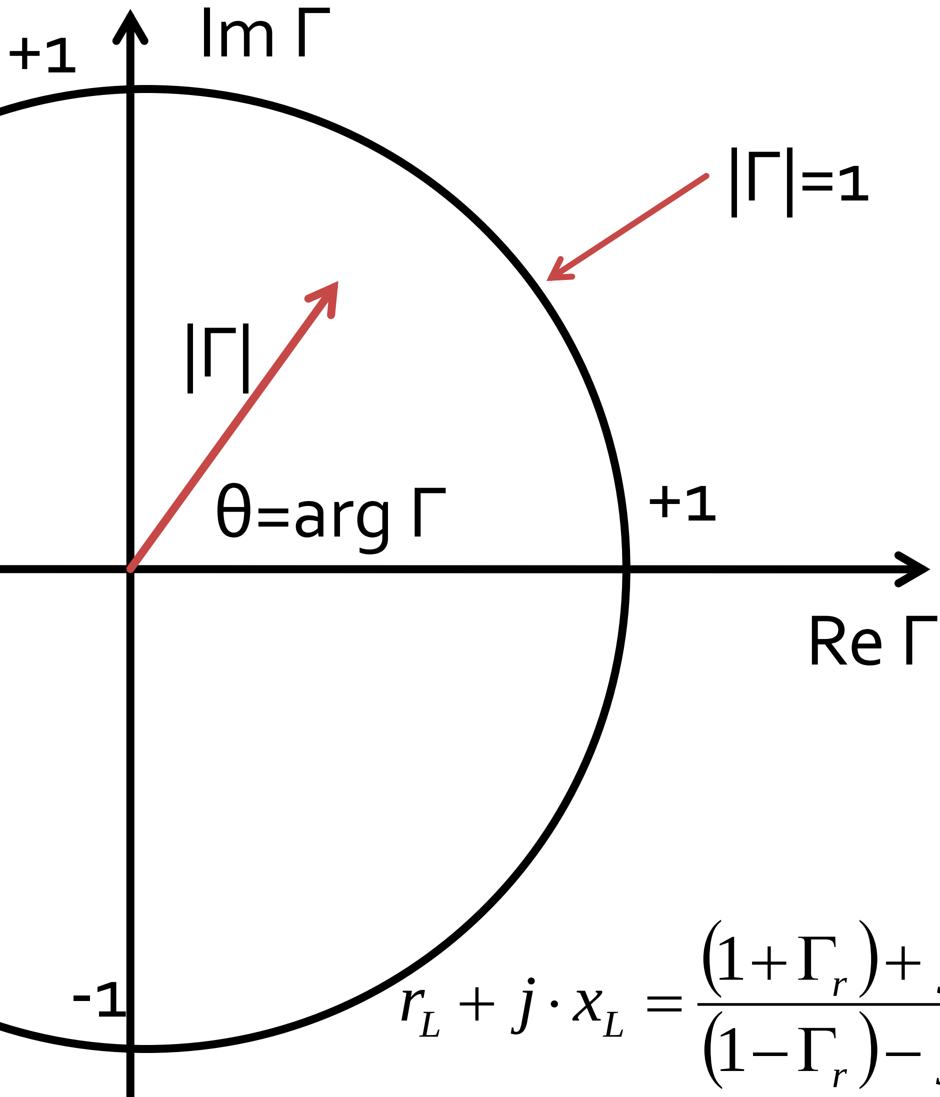
The Smith Chart

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{z_L - 1}{z_L + 1}$$



$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{Y_0 - Y_L}{Y_0 + Y_L} = \frac{1 - y_L}{1 + y_L}$$

The Smith Chart



$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{z_L - 1}{z_L + 1} = |\Gamma| \cdot e^{j\theta}$$

$$z_L = \frac{Z_L}{Z_0} \qquad y_L = \frac{Y_L}{Y_0} = \frac{Z_0}{Z_L}$$

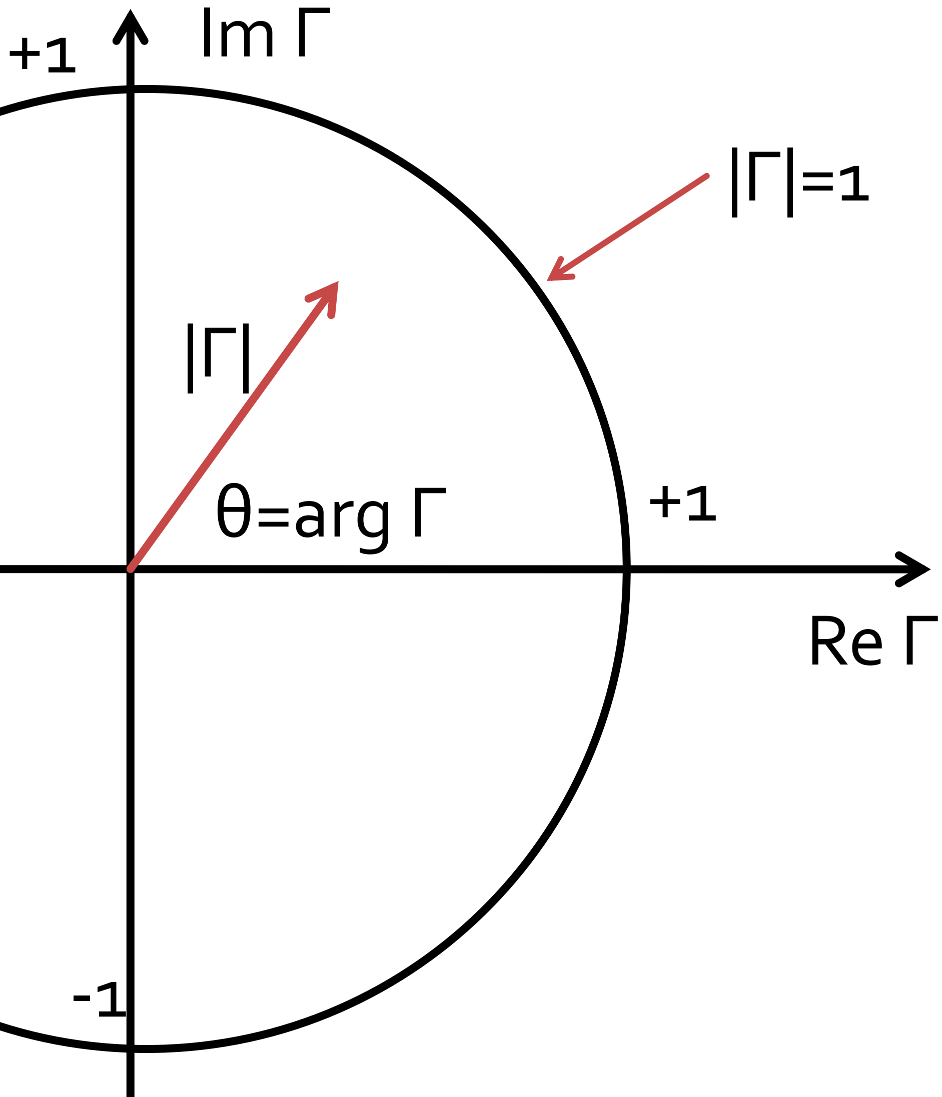
normalization $Z_L \rightarrow z_L$ allows using the same chart for any reference impedance Z_o (the plot becomes independent of the chosen Z_o)

$$\Gamma = \Gamma_r + j \cdot \Gamma_i$$

$$z_L = \frac{1 + |\Gamma| \cdot e^{j\theta}}{1 - |\Gamma| \cdot e^{j\theta}} = r_L + j \cdot x_L$$

$$r_L + j \cdot x_L = \frac{(1 + \Gamma_r) + j \cdot \Gamma_i}{(1 - \Gamma_r) - j \cdot \Gamma_i} = \frac{1 - \Gamma_r^2 - \Gamma_i^2}{(1 - \Gamma_r)^2 + \Gamma_i^2} + j \cdot \frac{2 \cdot \Gamma_i}{(1 - \Gamma_r)^2 + \Gamma_i^2}$$

The Smith Chart



$$r_L = \frac{1-\Gamma_r^2-\Gamma_i^2}{(1-\Gamma_r)^2+\Gamma_i^2}$$

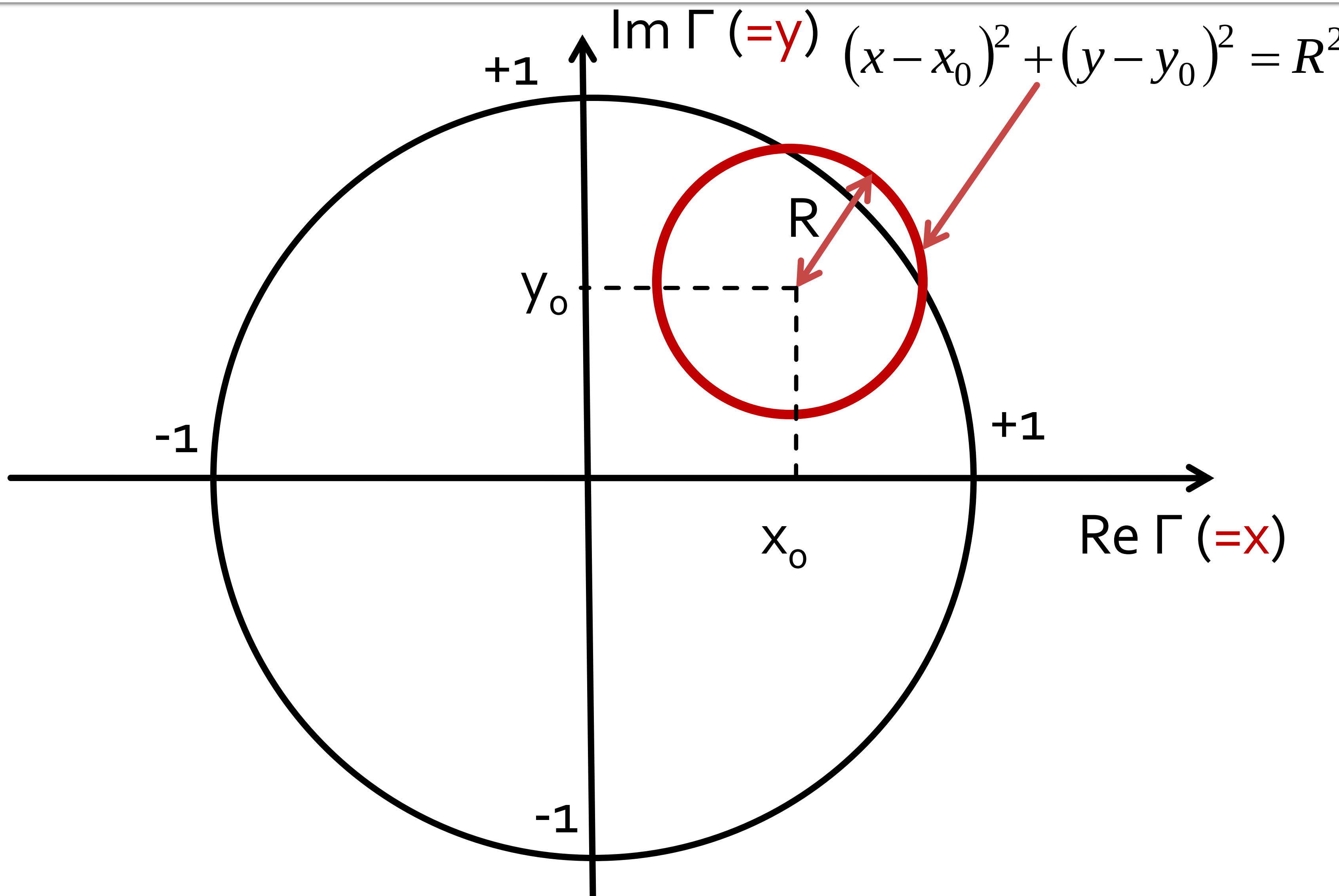
$$x_L = \frac{2\cdot\Gamma_i}{(1-\Gamma_r)^2+\Gamma_i^2}$$

- Rearranged

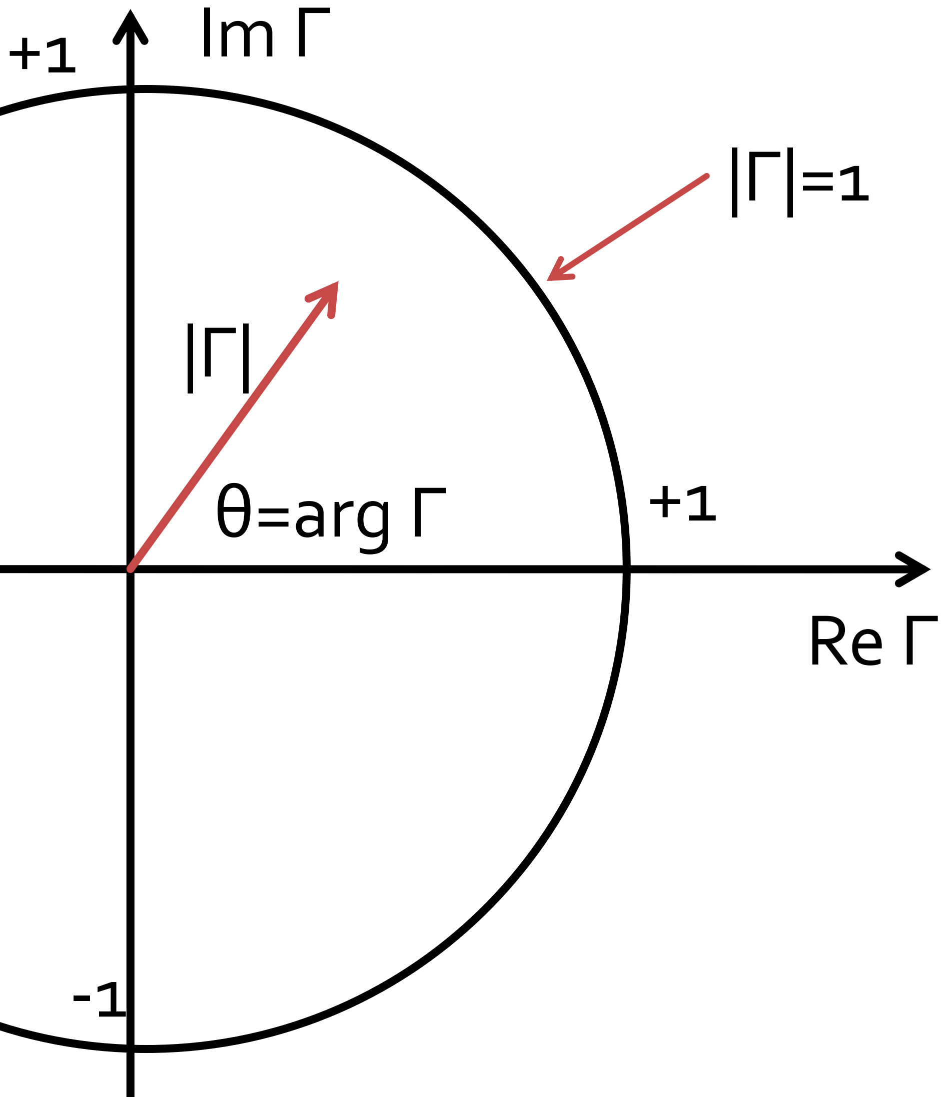
$$\left(\Gamma_r - \frac{r_L}{1+r_L}\right)^2 + \Gamma_i^2 = \left(\frac{1}{1+r_L}\right)^2$$

$$(\Gamma_r - 1)^2 + \left(\Gamma_i - \frac{1}{x_L}\right)^2 = \left(\frac{1}{x_L}\right)^2$$

The Smith Chart



The Smith Chart



$$\left(\Gamma_r - \frac{r_L}{1+r_L}\right)^2 + \Gamma_i^2 = \left(\frac{1}{1+r_L}\right)^2$$

$$\left(\Gamma_r - 1\right)^2 + \left(\Gamma_i - \frac{1}{x_L}\right)^2 = \left(\frac{1}{x_L}\right)^2$$

- **Circles** in the (Γ_r , Γ_i) complex plane

$$\left(x - x_0\right)^2 + \left(y - y_0\right)^2 = R^2$$

The Smith Chart, resistance

$$\left(\Gamma_r - \frac{r_L}{1+r_L}\right)^2 + \Gamma_i^2 = \left(\frac{1}{1+r_L}\right)^2$$

$$(x - x_0)^2 + (y - y_0)^2 = R^2$$

$$\begin{cases} x_0 = \dfrac{r_L}{1+r_L} \\ y_0 = 0 \\ R = \dfrac{1}{1+r_L} \end{cases}$$

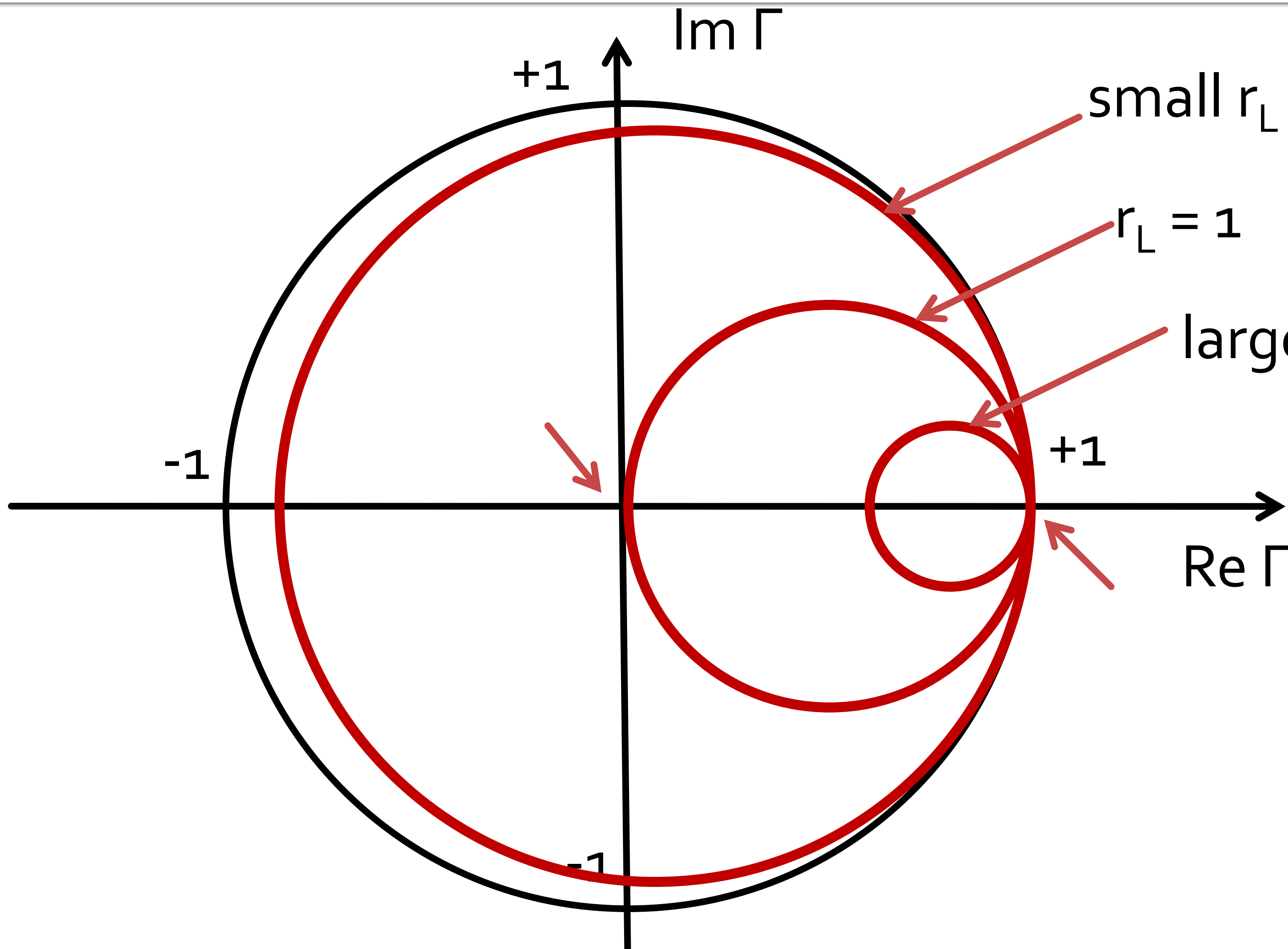
- The locus (the set of all points whose location satisfies one or more specified conditions) of the points generated by all impedances having normalized resistance r_L is a circle which:
  - have its **center on the horizontal axis** (y_o=o)
  - passes through **x=1,y=o** point, whatever x_o,r_L

    $$\left(1 - \frac{r_L}{1+r_L}\right)^2 + 0 = \left(\frac{1}{1+r_L}\right)^2$$

  - have its radius between o and 1
    - tends to o for large r_L
    - tends to 1 for small r_L
  - when r_L is **1** passes also through **origin**

    $$\left(0 - \frac{r_L}{1+r_L}\right)^2 = \left(\frac{1}{1+r_L}\right)^2 \leftrightarrow r_L = 1$$

The Smith Chart, resistance



The Smith Chart, reactance

$$(\Gamma_r - 1)^2 + \left(\Gamma_i - \frac{1}{x_L}\right)^2 = \left(\frac{1}{x_L}\right)^2$$

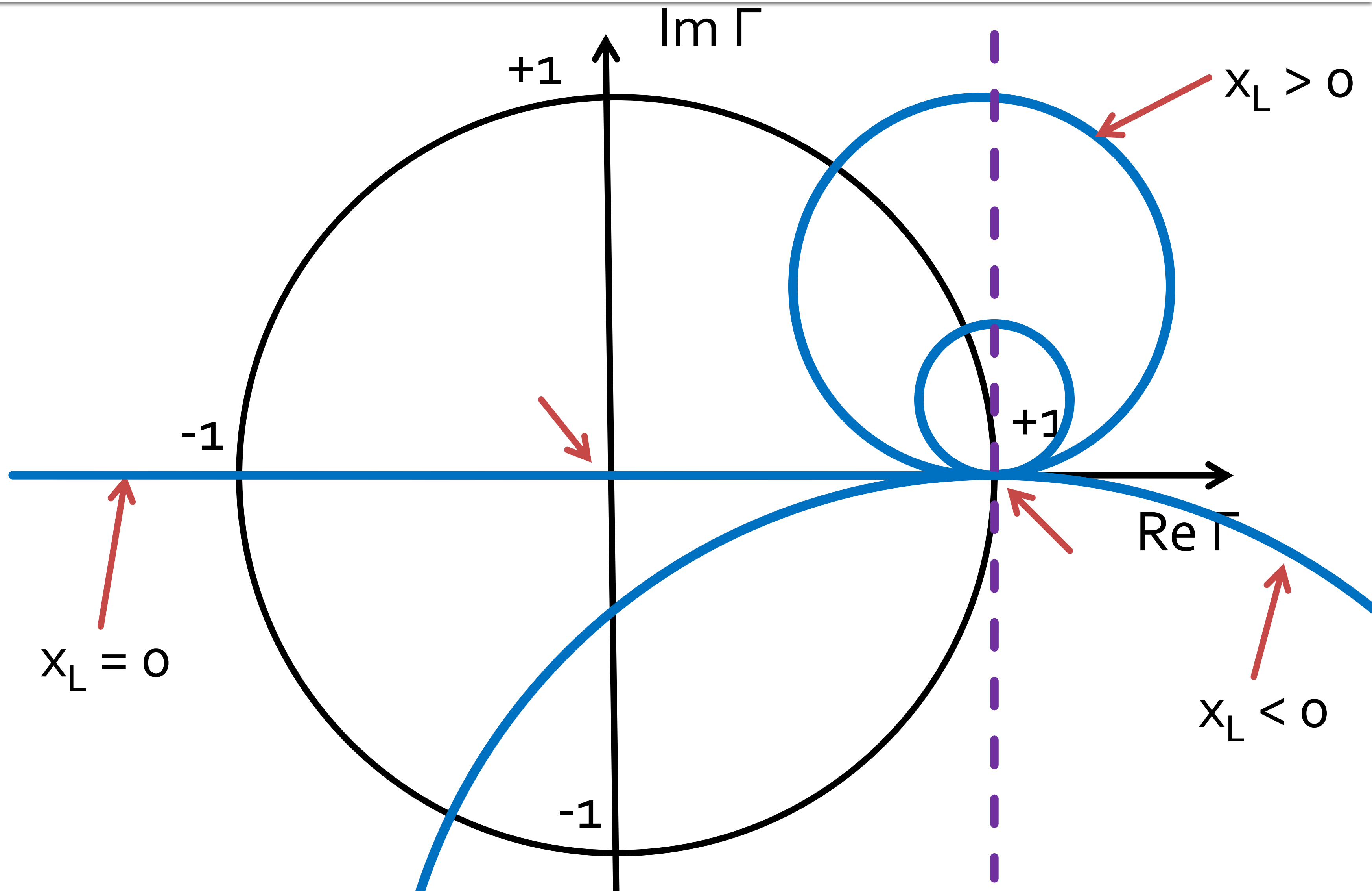
$$(x - x_0)^2 + (y - y_0)^2 = R^2$$

$$\begin{cases} x_0 = 1 \\ y_0 = \dfrac{1}{x_L} \\ R = \dfrac{1}{x_L} \end{cases}$$

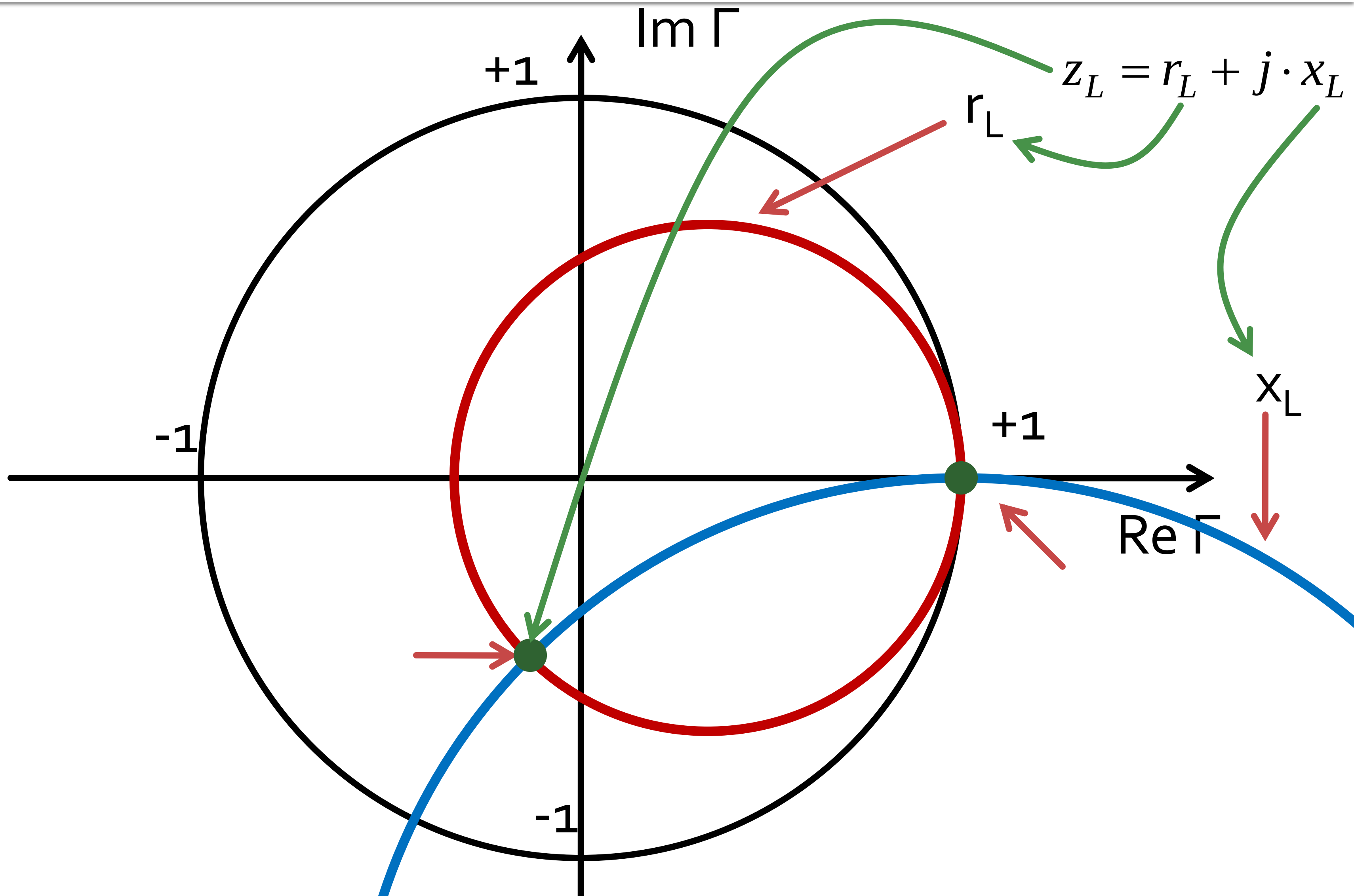
- The locus of the points generated by all impedances having normalized resistance x_L is a circle which:
  - have its **center on a line parallel with the vertical axis** (x_0=1)
  - passes through **x=1,y=0** point, whatever x_0,x_L
  - have its radius between 0 and ∞
    - tends to 0 for large $|x_L|$
    - tends to ∞ for small $|x_L|$
  - when x_L is **0** transforms itself in the **horizontal axis**
  - if x_L > 0 the circle is above the horizontal axis, otherwise is bellow it

$$0 + \left(0 - \frac{1}{x_L}\right)^2 = \left(\frac{1}{x_L}\right)^2$$

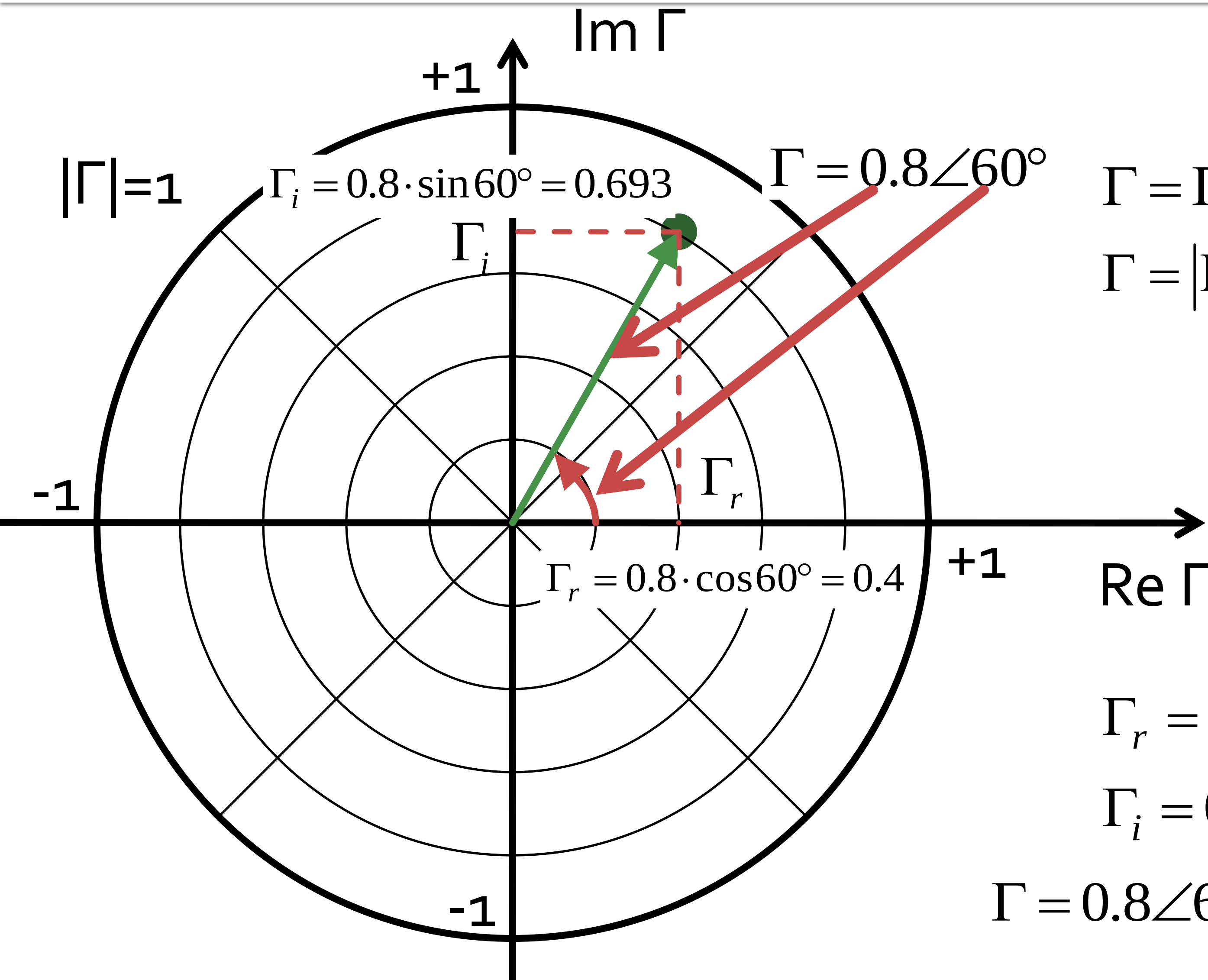
The Smith Chart, reactance



The Smith Chart, impedance



The Smith Chart, reflection coefficient, Cartesian coordinate system



$$\Gamma = \Gamma_r + j \cdot \Gamma_i$$

$$\Gamma = |\Gamma| \cdot (\cos\theta + j \cdot \sin\theta)$$

$$\Gamma = |\Gamma| \cdot e^{j\theta}$$

$$\Gamma = |\Gamma| \angle \theta°$$

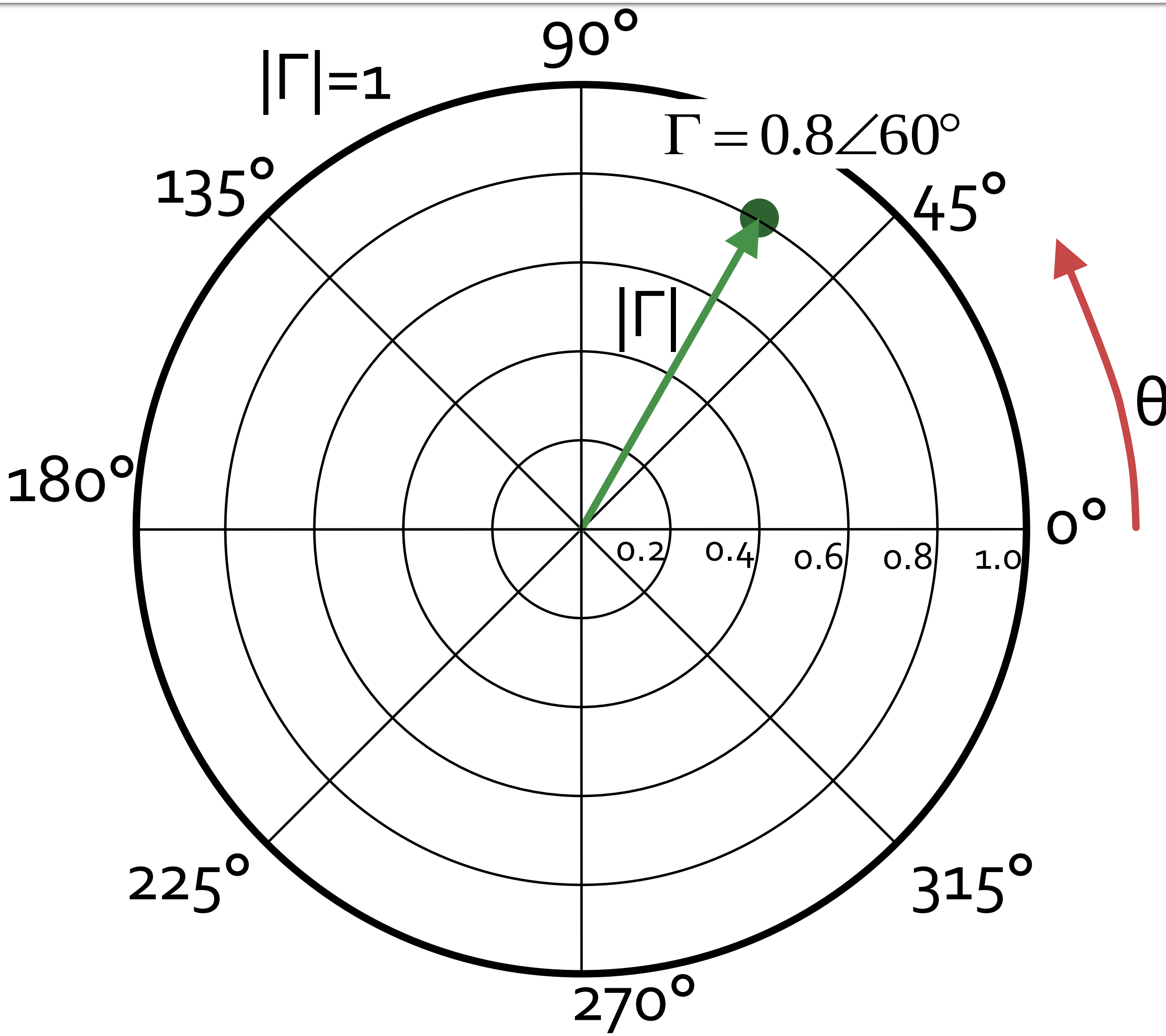
$$\Gamma = 0.8\angle 60°$$

$$\Gamma_r = 0.8 \cdot \cos 60° = 0.4$$

$$\Gamma_i = 0.8 \cdot \sin 60° = 0.693$$

$$\Gamma = 0.8\angle 60° = 0.4 + j \cdot 0.693$$

The Smith Chart, reflection coefficient, Polar coordinate system



$$\Gamma = \Gamma_r + j \cdot \Gamma_i$$

$$\Gamma = |\Gamma| \cdot (\cos\theta + j \cdot \sin\theta)$$

$$\Gamma = |\Gamma| \cdot e^{j\theta}$$

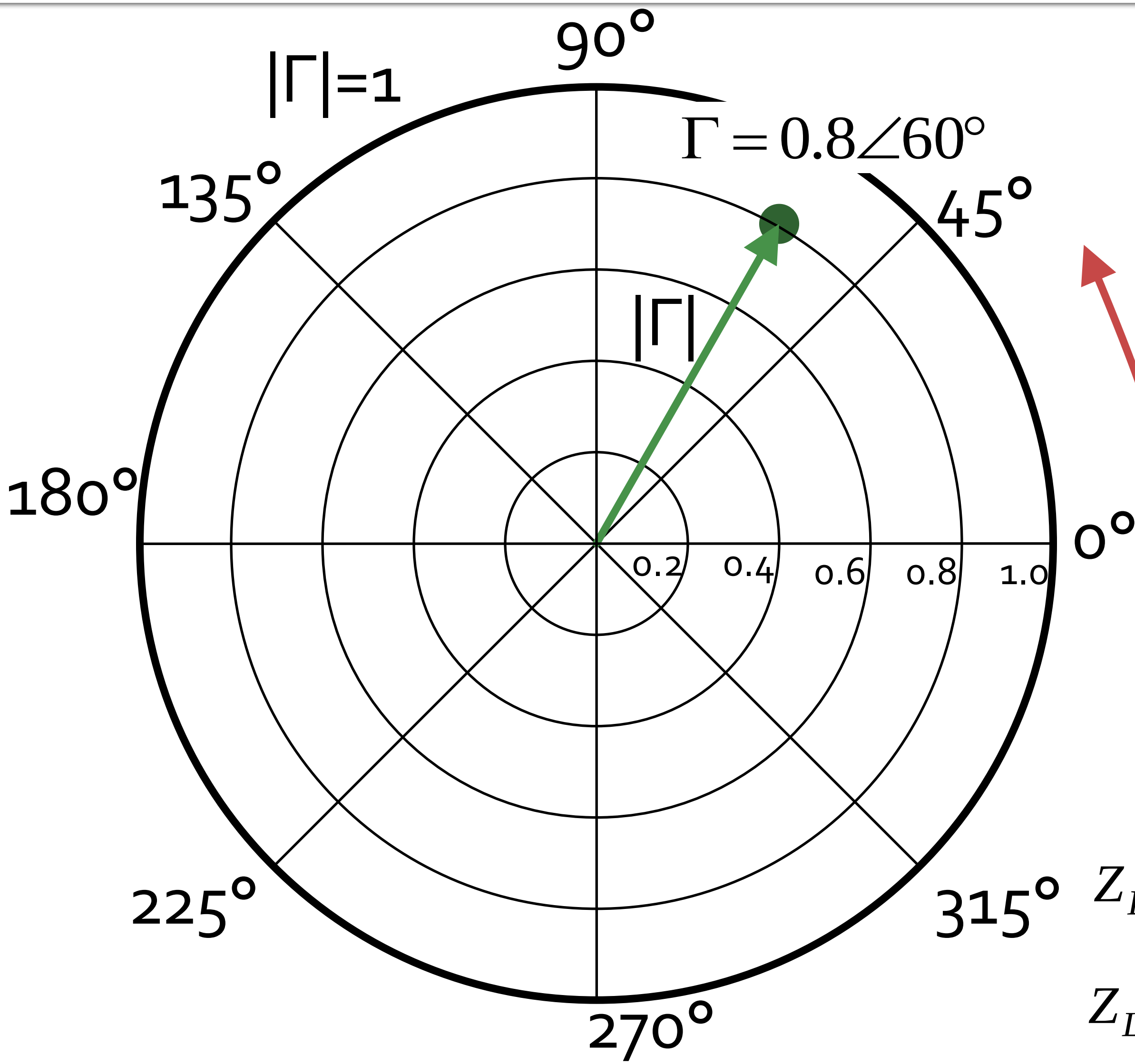
$$\Gamma = |\Gamma| \angle \theta^\circ$$

$$\Gamma = 0.8 \angle 60^\circ$$

$$\Gamma_r = 0.8 \cdot \cos 60^\circ = 0.4$$

$$\Gamma_i = 0.8 \cdot \sin 60^\circ = 0.693$$

The Smith Chart, reflection coefficient, impedance



$$\Gamma = |\Gamma| \cdot e^{j\theta}$$

$$\Gamma = |\Gamma| \angle \theta°$$

$$\Gamma = 0.8\angle 60°$$

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{z_L - 1}{z_L + 1}$$

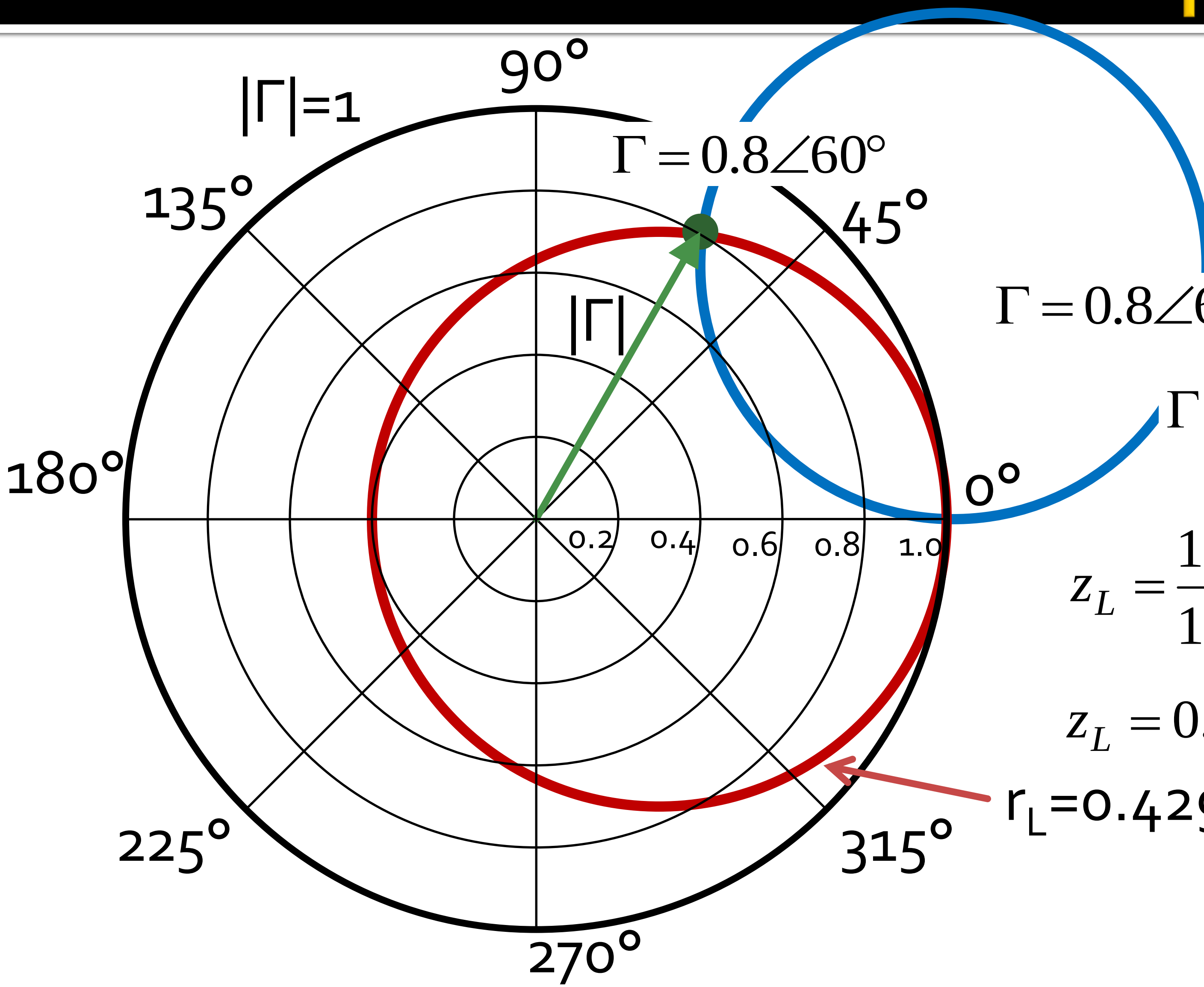
$$z_L = \frac{1+\Gamma}{1-\Gamma} = \frac{1+0.8\angle 60°}{1-0.8\angle 60°}$$

$$z_L = 0.429 + j \cdot 1.65$$

$$Z_L = Z_0 \cdot \frac{1+\Gamma}{1-\Gamma} = 50\Omega \cdot \frac{1+0.8\angle 60°}{1-0.8\angle 60°}$$

$$Z_L = 21.429\Omega + j \cdot 82.479\Omega$$

Equivalence
reflection coefficient ⇔ impedance



$$\Gamma = 0.8\angle 60° = 0.4 + j \cdot 0.693$$

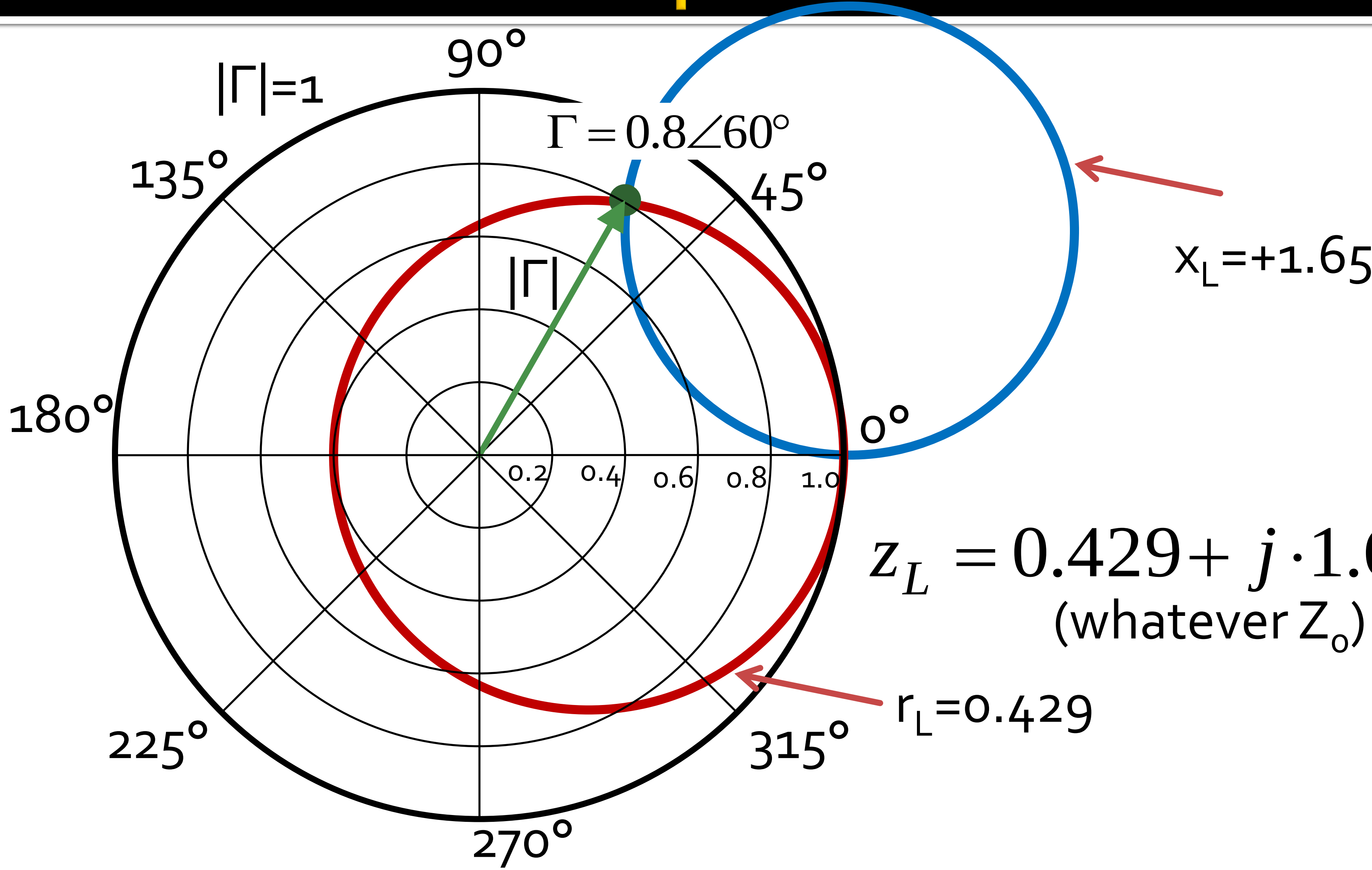
$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{z_L - 1}{z_L + 1}$$

$$z_L = \frac{1+\Gamma}{1-\Gamma} = \frac{1+0.8\angle 60°}{1-0.8\angle 60°}$$

$$z_L = 0.429 + j \cdot 1.65$$

(whatever Z_o)

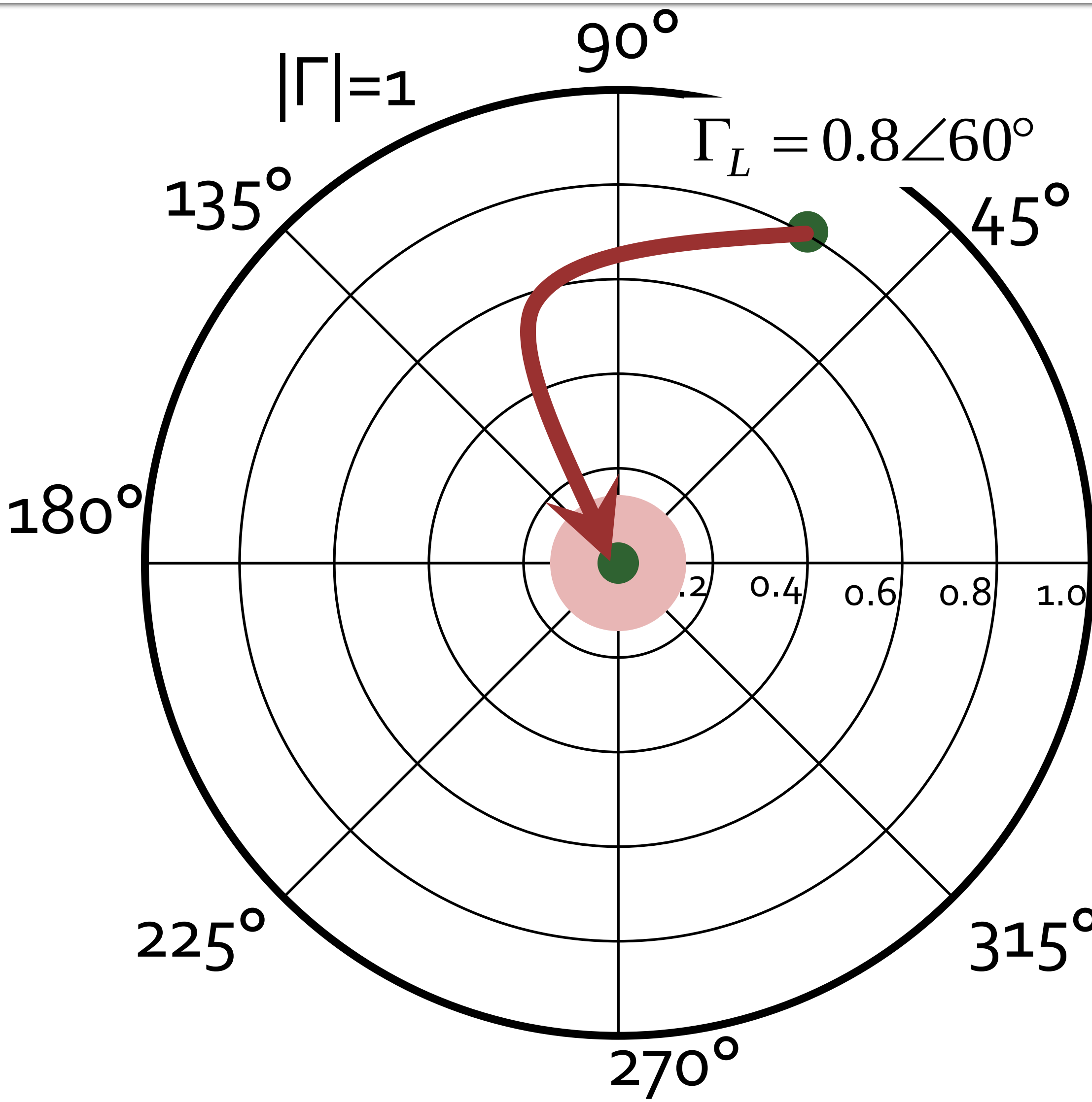
The Smith Chart, reflection coefficient ⇔ impedance

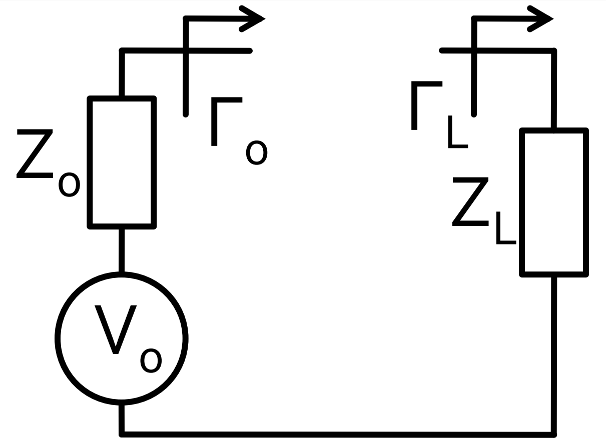


$$z_L = 0.429 + j \cdot 1.65$$

(whatever Z_o)

The Smith Chart, reflection coefficient, matching





Matching Z_L load to Z_o source.
We normalize Z_L over Z_o

$$Z_L = 21.429\Omega + j \cdot 82.479\Omega$$

$$z_L = 0.429 + j \cdot 1.65$$

$$\Gamma_L = 0.8\angle 60°$$

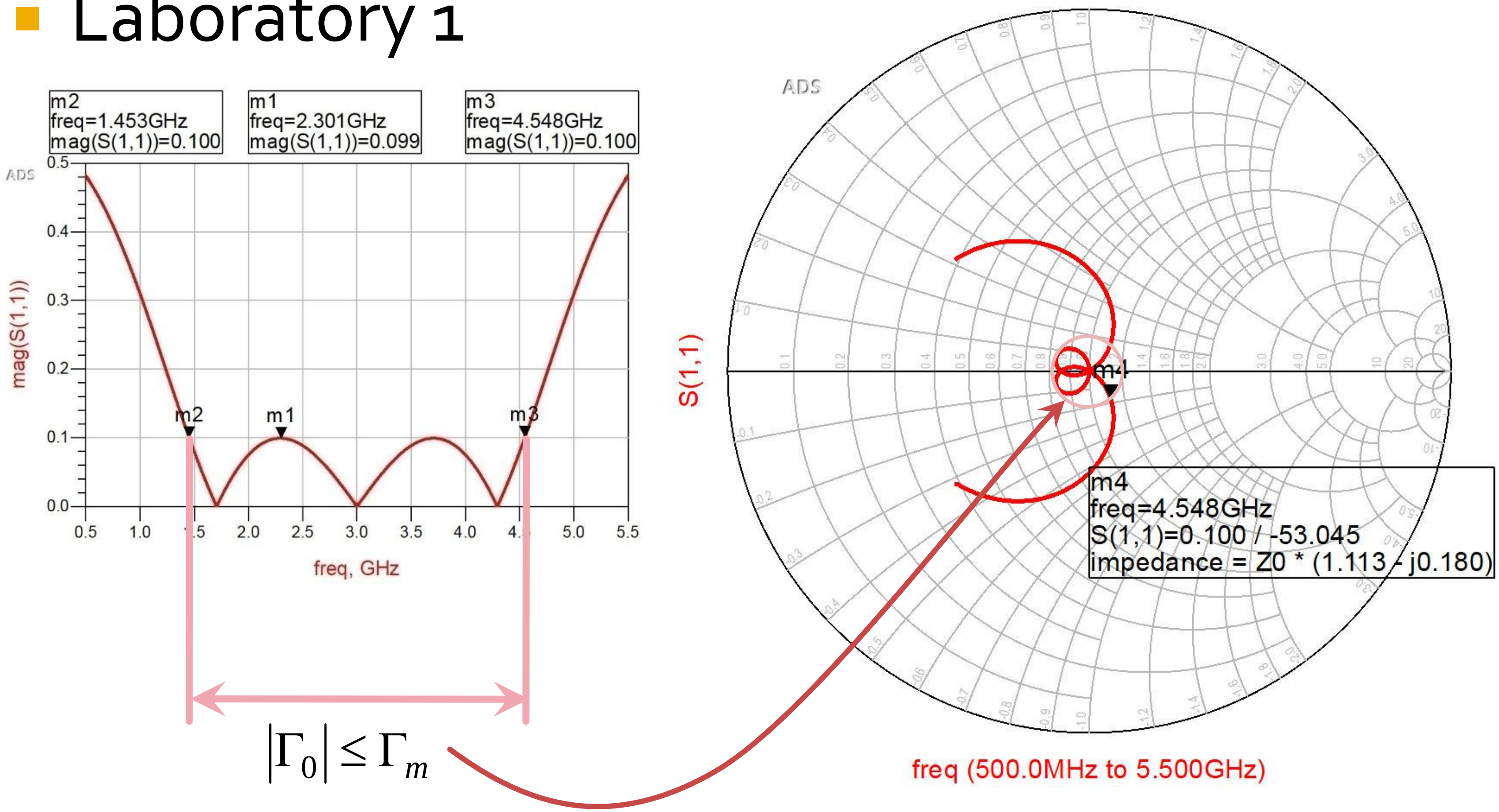
We must move the point denoting the reflection coefficient in the area where with a Z_o source we have:

$\Gamma_0 = 0$ perfect match

$|\Gamma_0| \leq \Gamma_m$ “good enough” match

Example

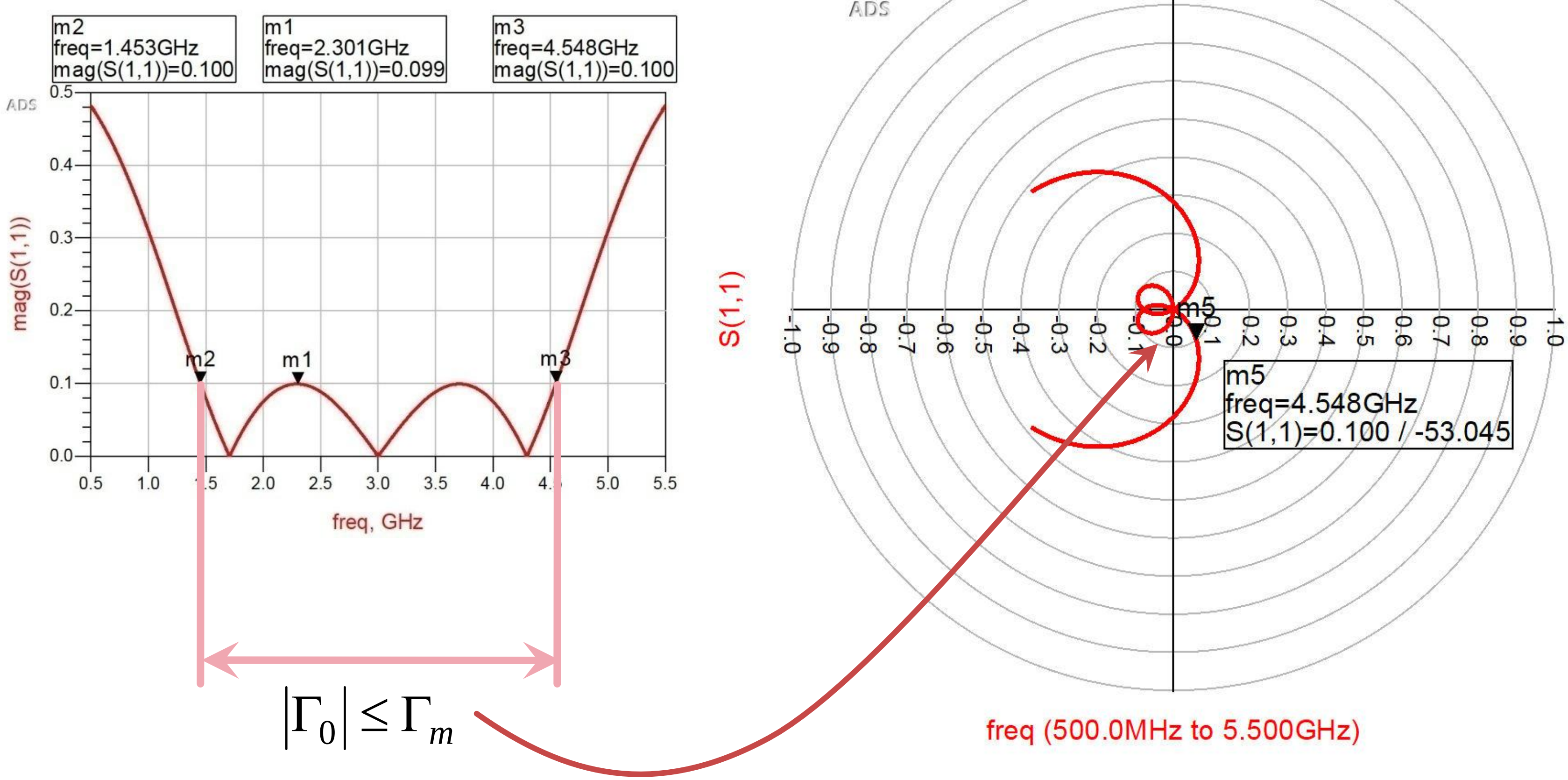
- Laboratory 1



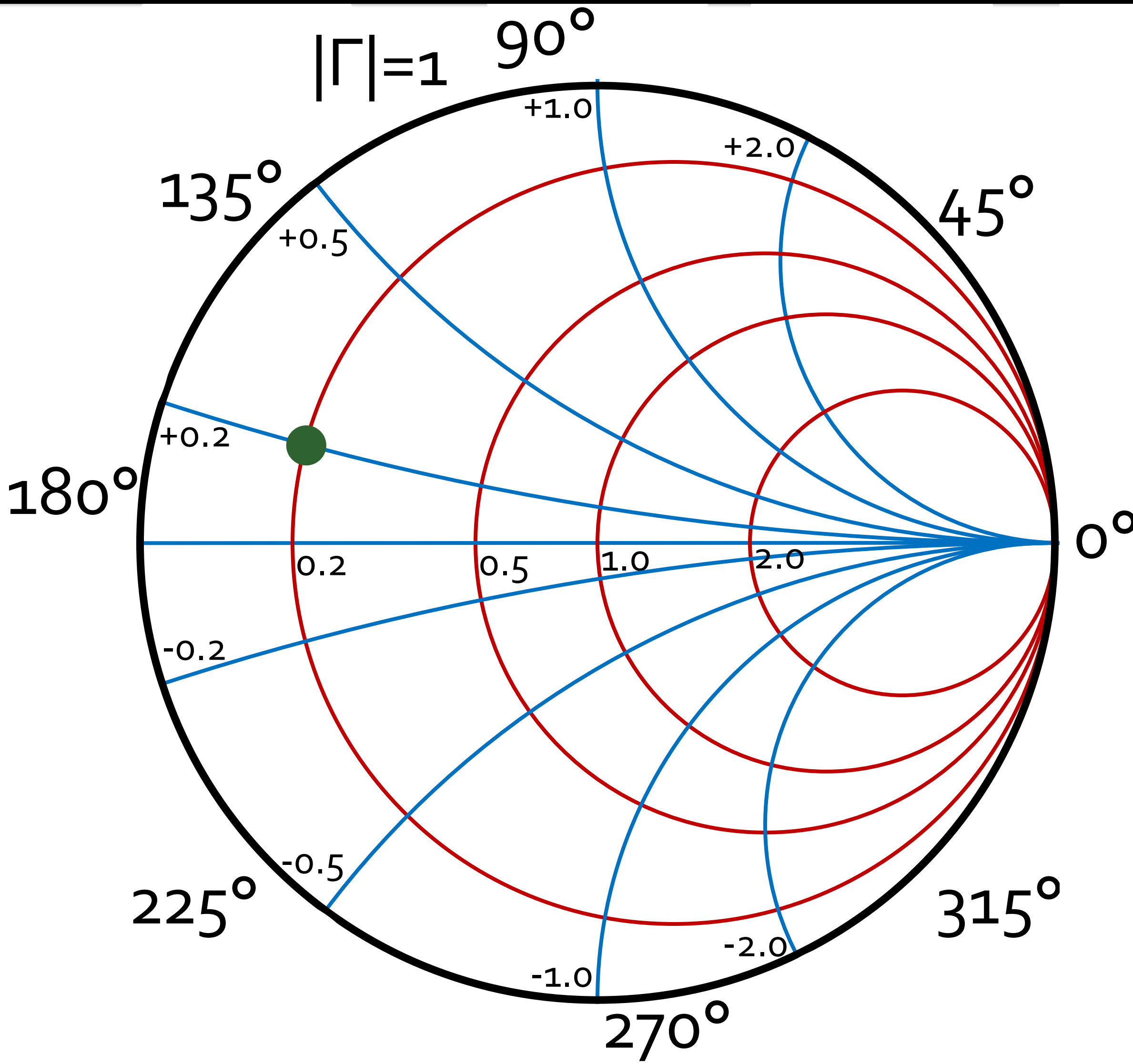
$$\left|\Gamma_0\right| \leq \Gamma_m$$

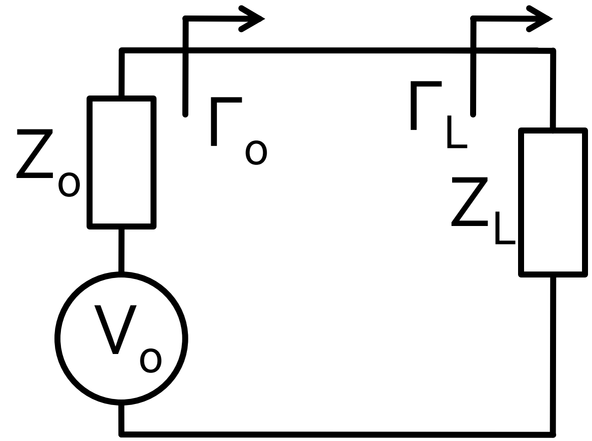
Example

- Laboratory 1



The Smith Chart, impedance/reflection coefficient





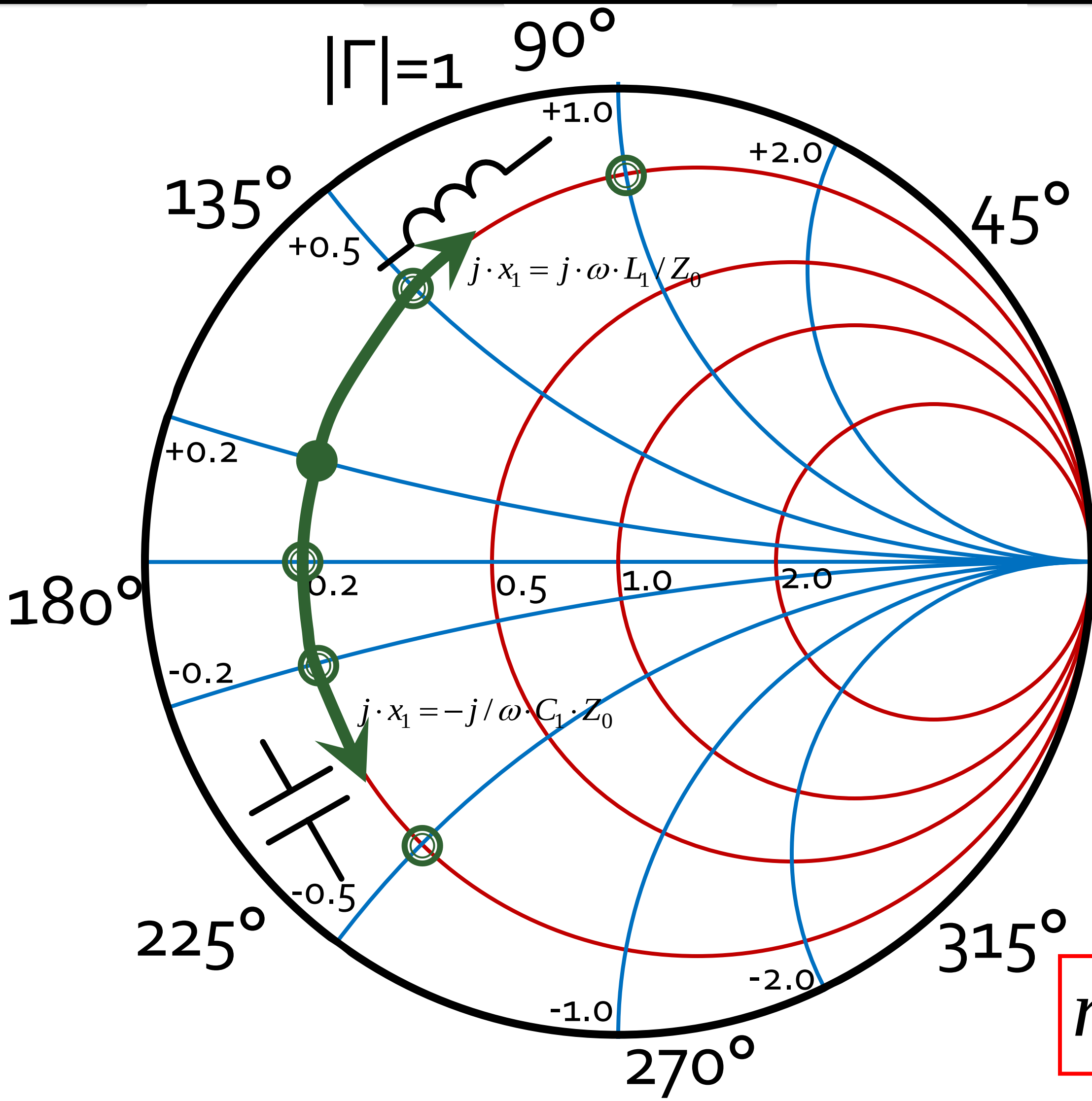
$Z_0 = 50\Omega$

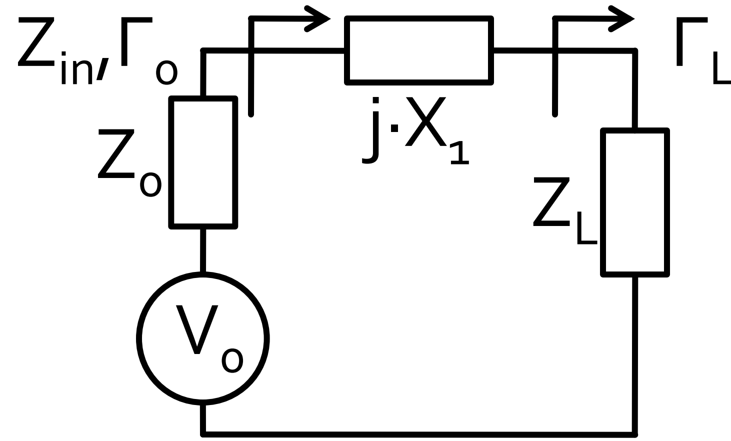
$Z_L = 10\Omega + j \cdot 10\Omega$

$z_L = 0.2 + j \cdot 0.2$

$\Gamma_L = \Gamma_0 = 0.678\angle 156.5$

The Smith Chart, series reactance





$$Z_0 = 50\Omega$$

$$Z_L = R_L + j \cdot X_L = 10\Omega + j \cdot 10$$

$$z_L = r_L + j \cdot x_L = 0.2 + j \cdot 0.2$$

$$\Gamma_L = 0.678\angle 156.5°$$

$$Z_{in} = Z_L + j \cdot X_1 = R_L + j \cdot (X_L + X_1$$

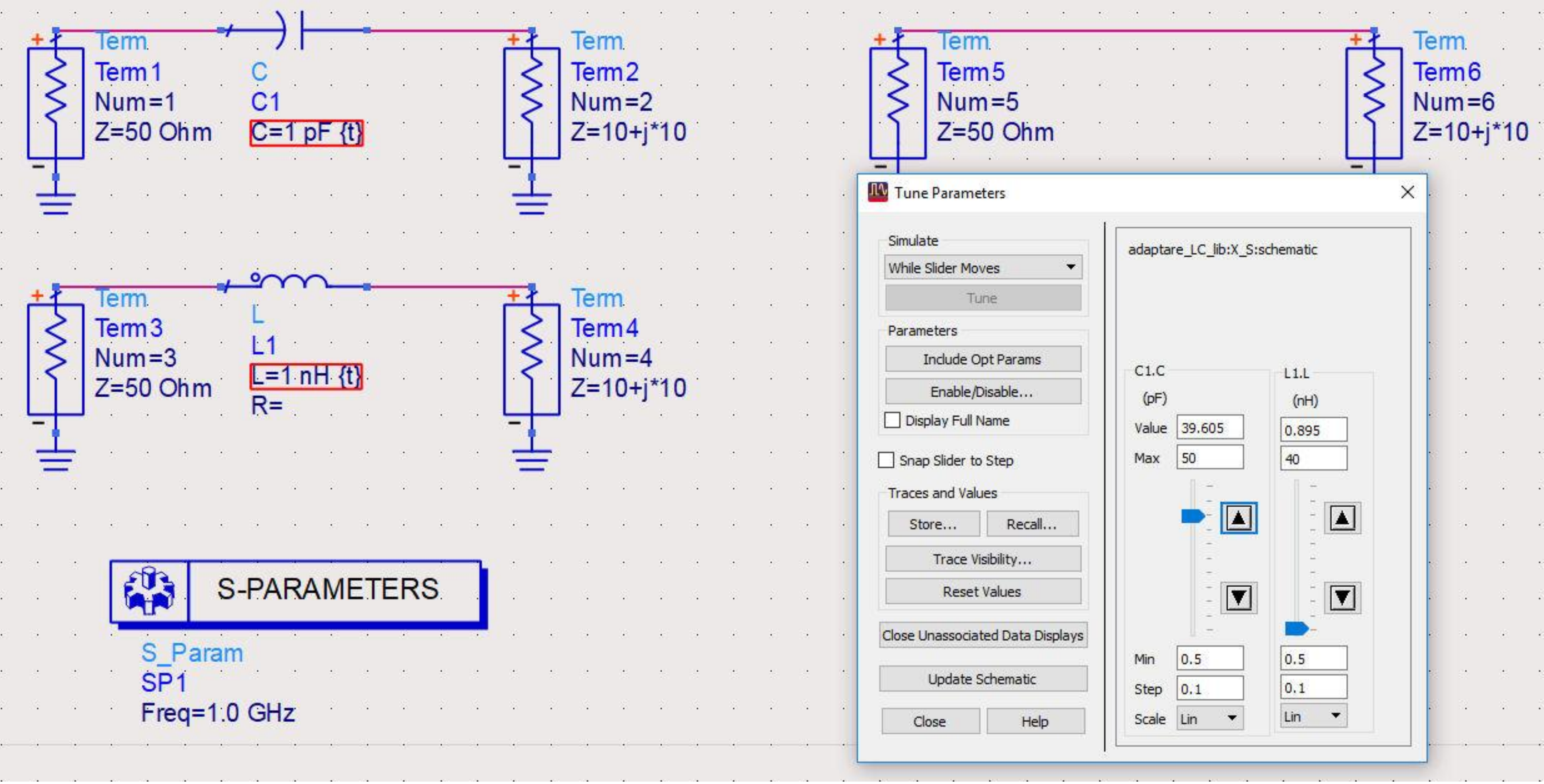
$$z_{in} = r_L + j \cdot (x_L + x_1)$$

$$r_{in} = r_L$$

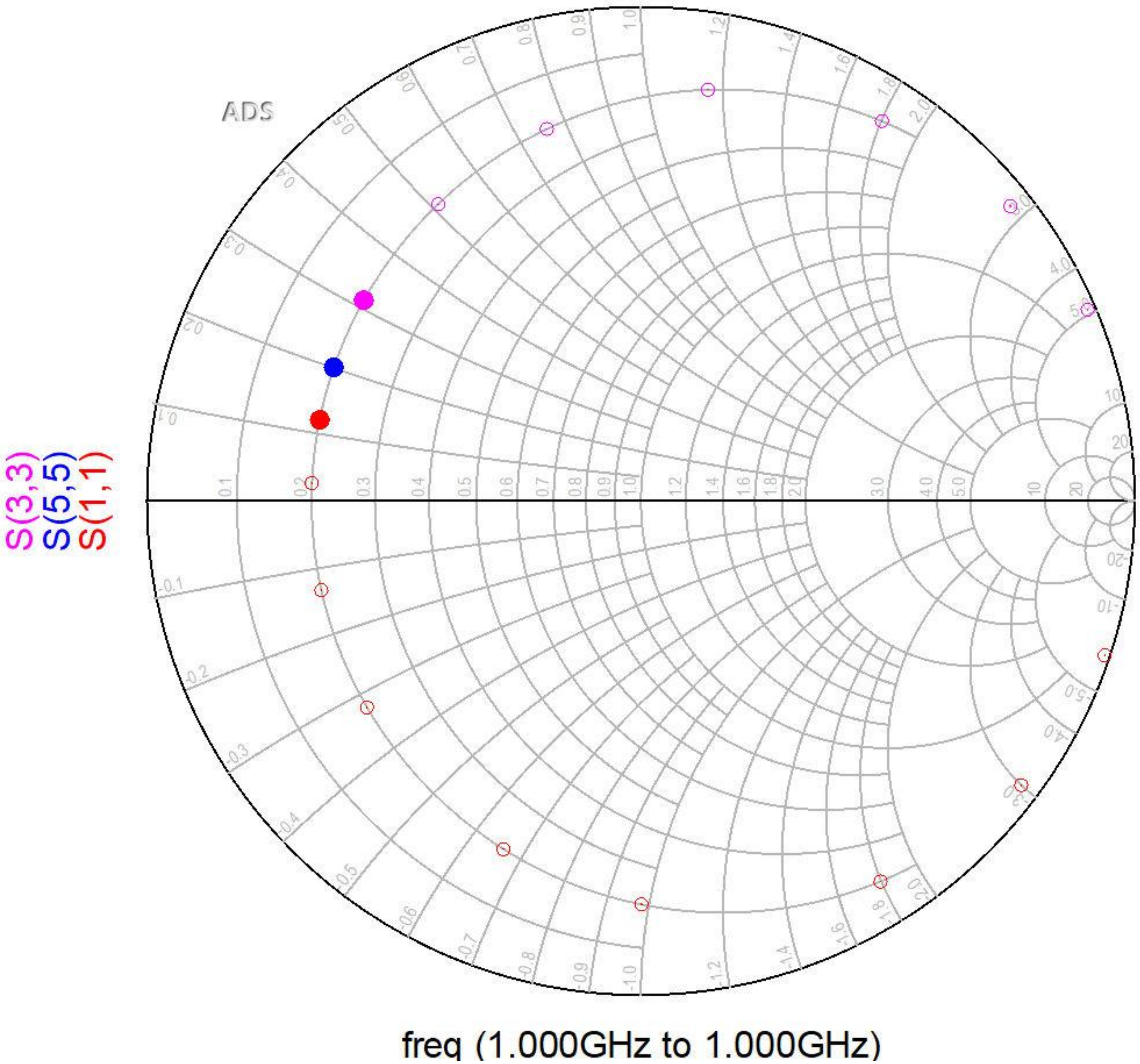
$$j \cdot x_1 = j \cdot \omega \cdot L_1 / Z_0 > 0$$

$$j \cdot x_1 = -j / \omega \cdot C_1 \cdot Z_0 < 0$$

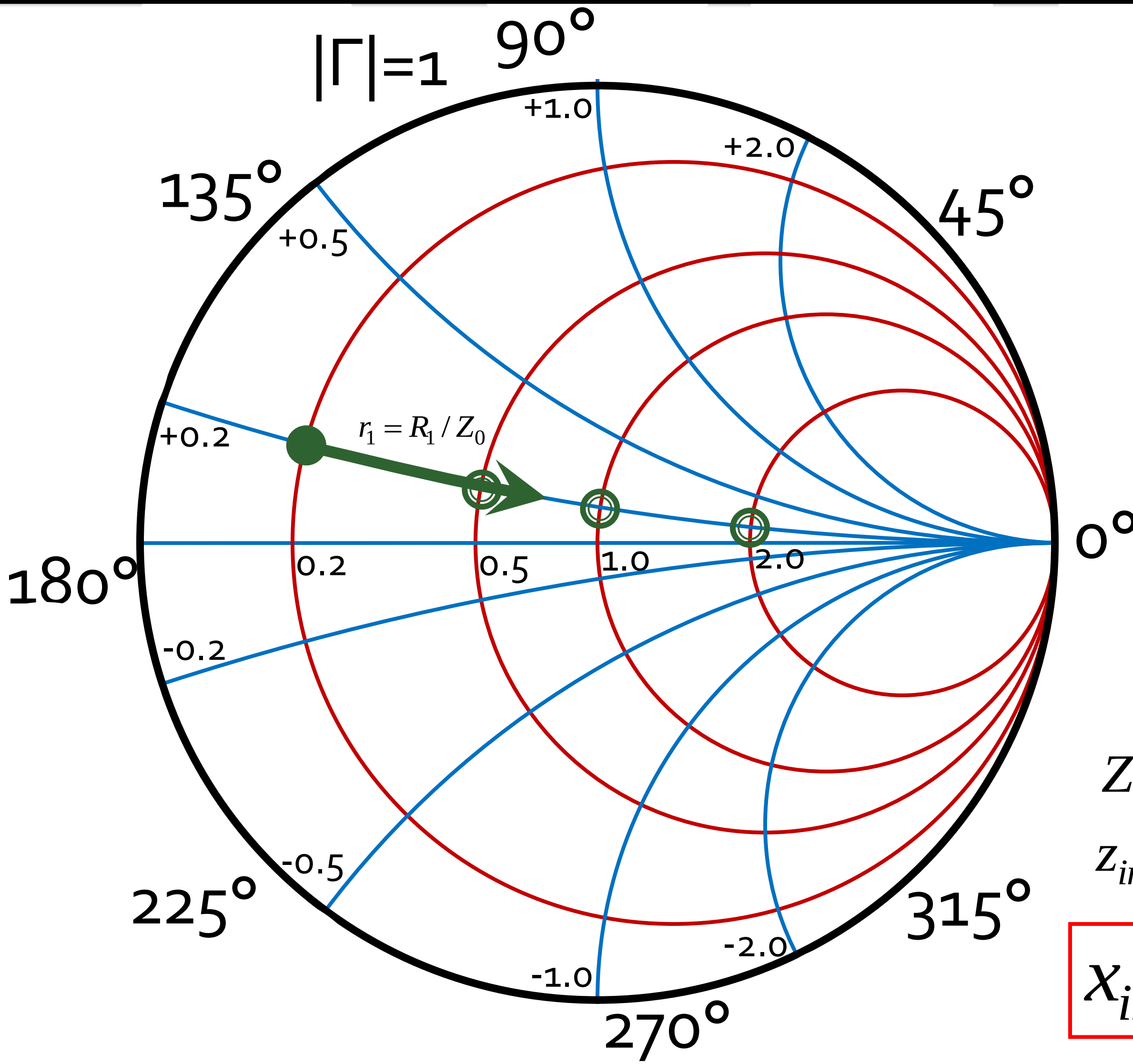
ADS, Smith Chart, series reactance

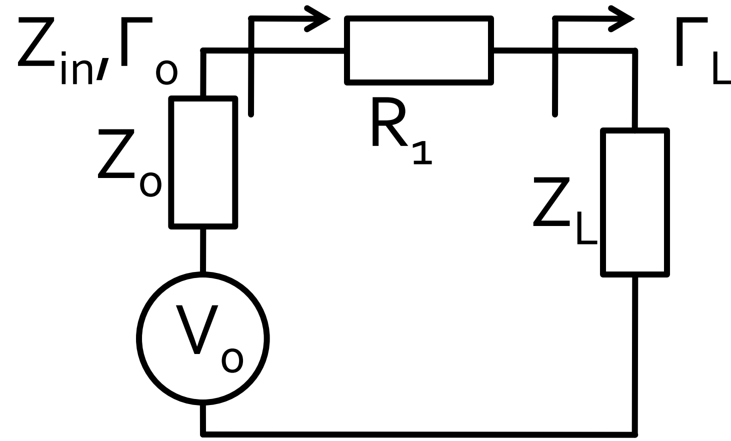


ADS, Smith Chart, series reactance



The Smith Chart, series resistance





$$Z_0 = 50\Omega$$

$$Z_L = R_L + j\cdot X_L = 10\Omega + j\cdot 10$$

$$z_L = r_L + j\cdot x_L = 0.2 + j\cdot 0.2$$

$$\Gamma_L = 0.678\angle 156.5°$$

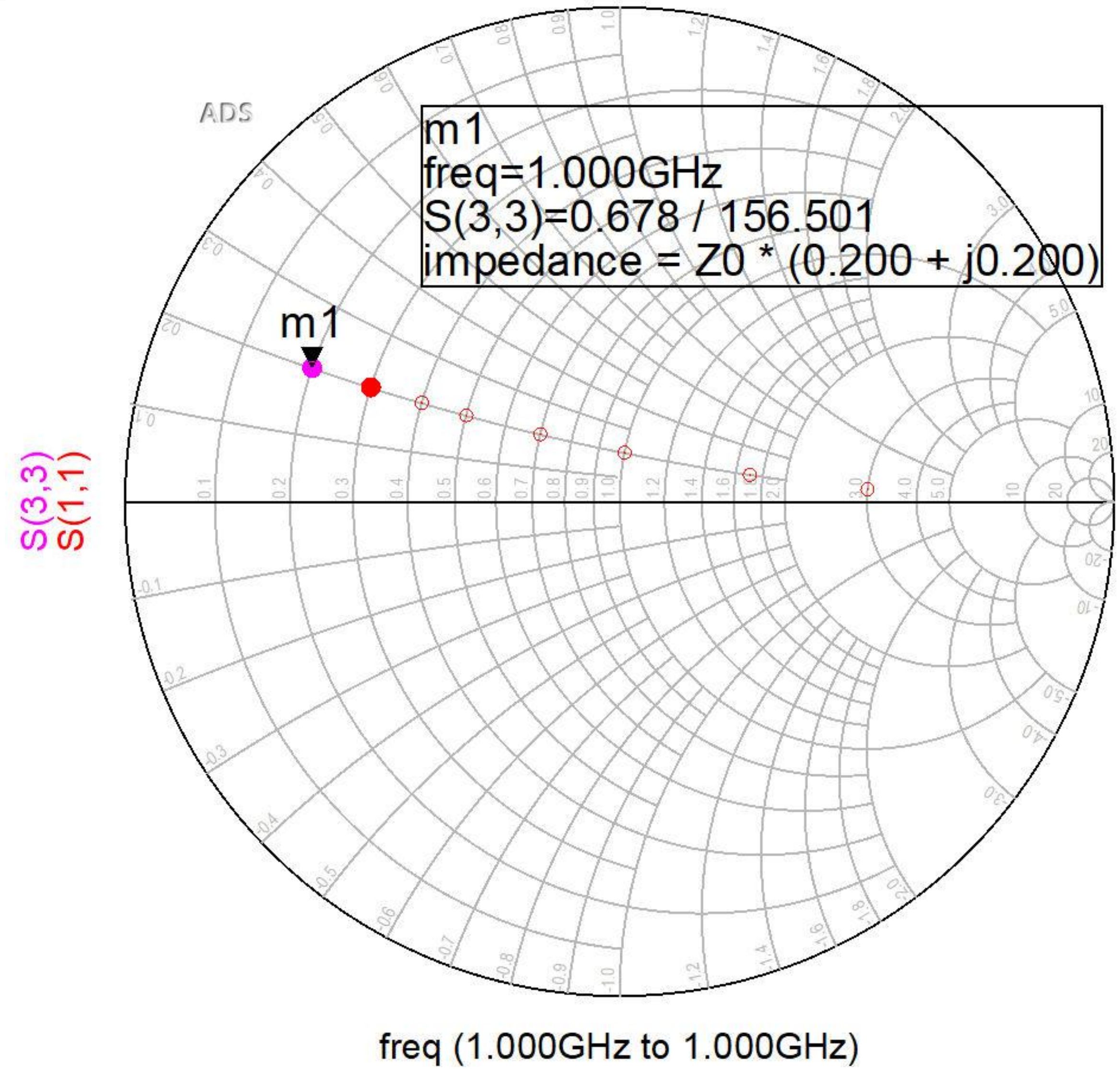
$$Z_{in} = Z_L + R_1 = (R_L + R_1) + j\cdot X_L$$

$$z_{in} = z_L + r_1 = (r_L + r_1) + j\cdot x_L$$

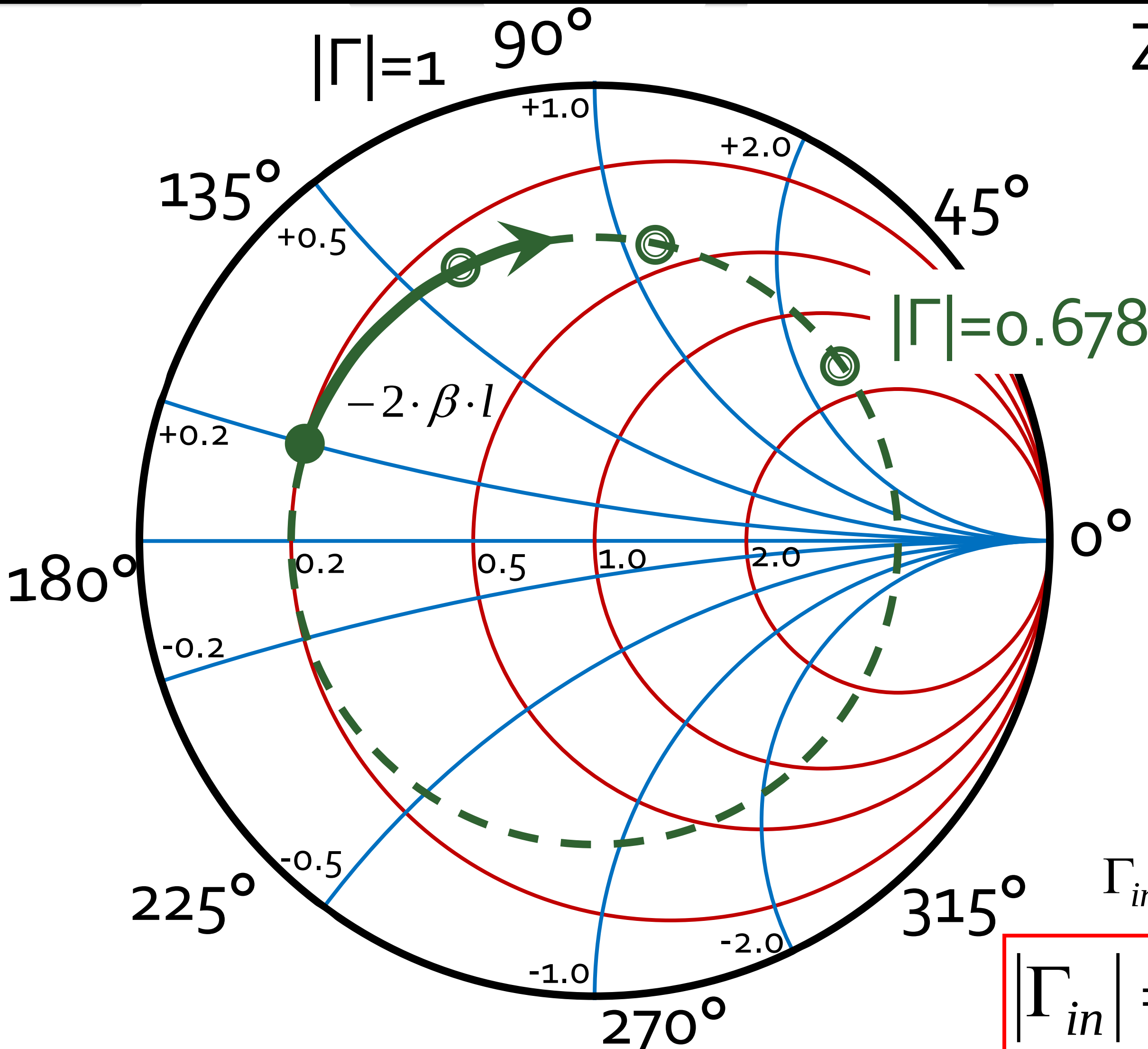
$$x_{in} = x_L$$

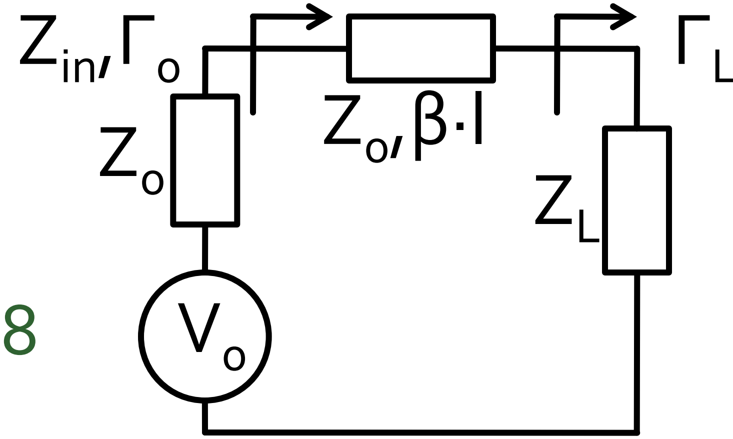
$$r_{in} = r_L + R_1 / Z_0$$

ADS, Smith Chart, series resistance



The Smith Chart, series transmission line, Z_o





$$Z_0 = 50\Omega$$

$$Z_L = R_L + j\cdot X_L = 10\Omega + j\cdot 10$$

$$z_L = r_L + j\cdot x_L = 0.2 + j\cdot 0.2$$

$$\Gamma_L = 0.678\angle 156.5°$$

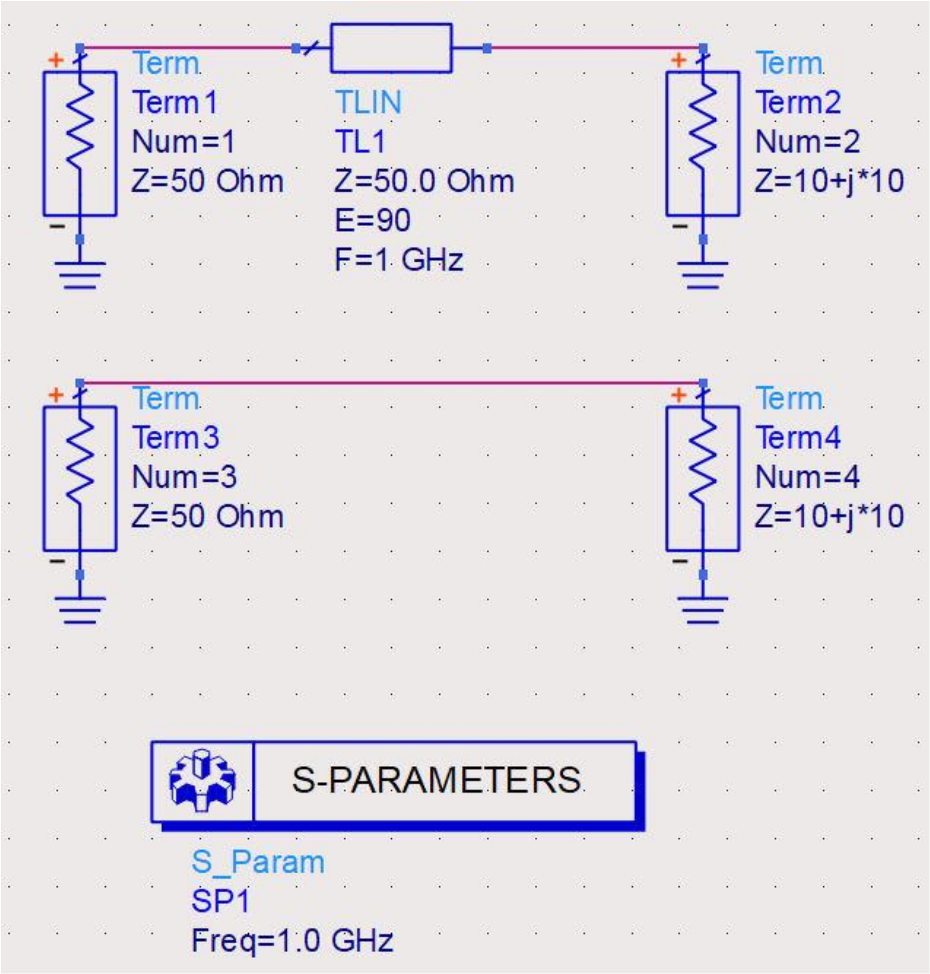
$$Z_{in} = Z_0 \cdot \frac{1+\Gamma_L\cdot e^{-2j\cdot\beta\cdot l}}{1-\Gamma_L\cdot e^{-2j\cdot\beta\cdot l}}$$

$$\Gamma_{in} = \Gamma_L\cdot e^{-2j\cdot\beta\cdot l}$$

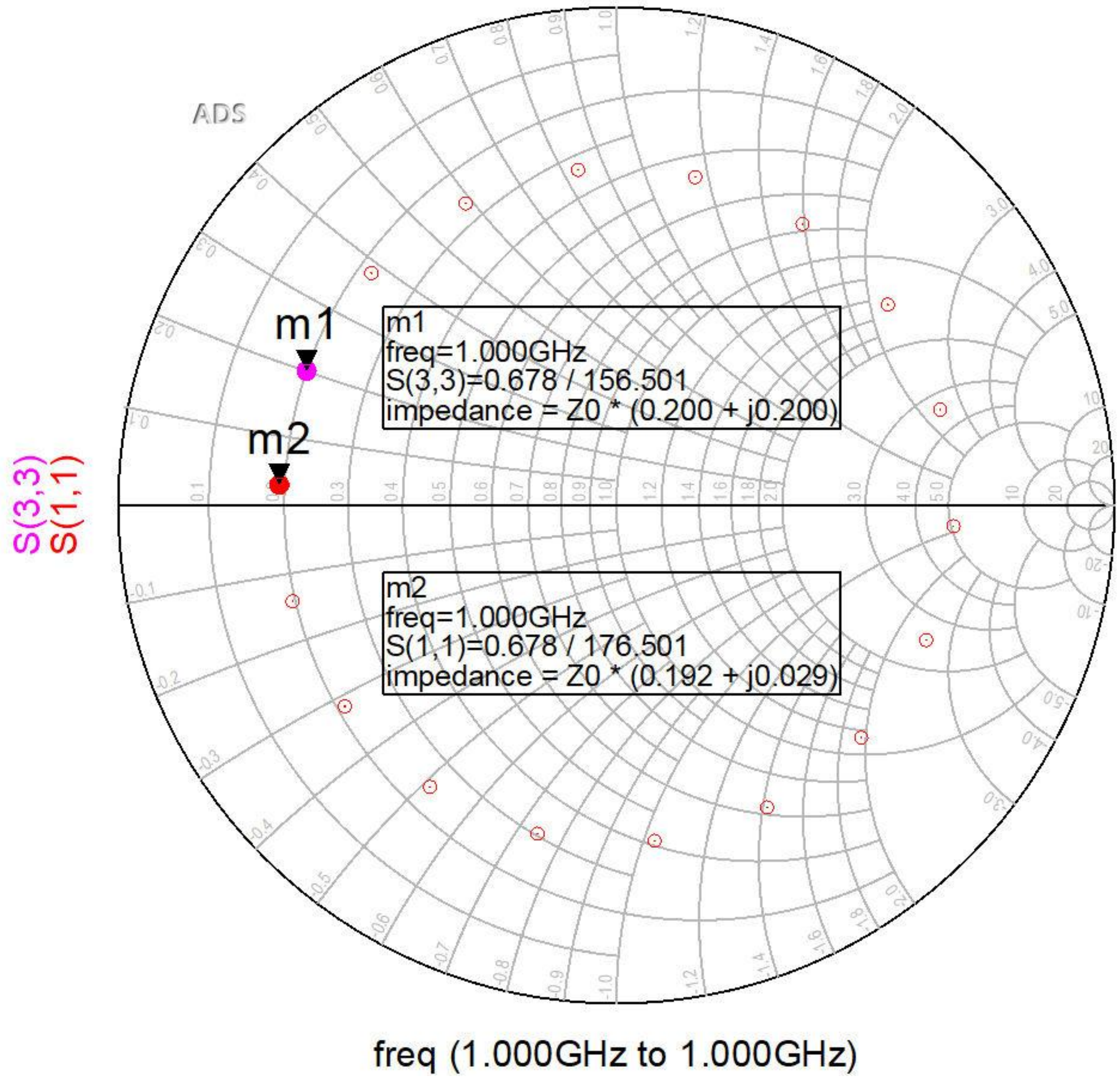
$$\left|\Gamma_{in}\right| = \left|\Gamma_L\right|$$

$$\arg(\Gamma_{in}) = \arg(\Gamma_L) - 2\cdot\beta\cdot$$

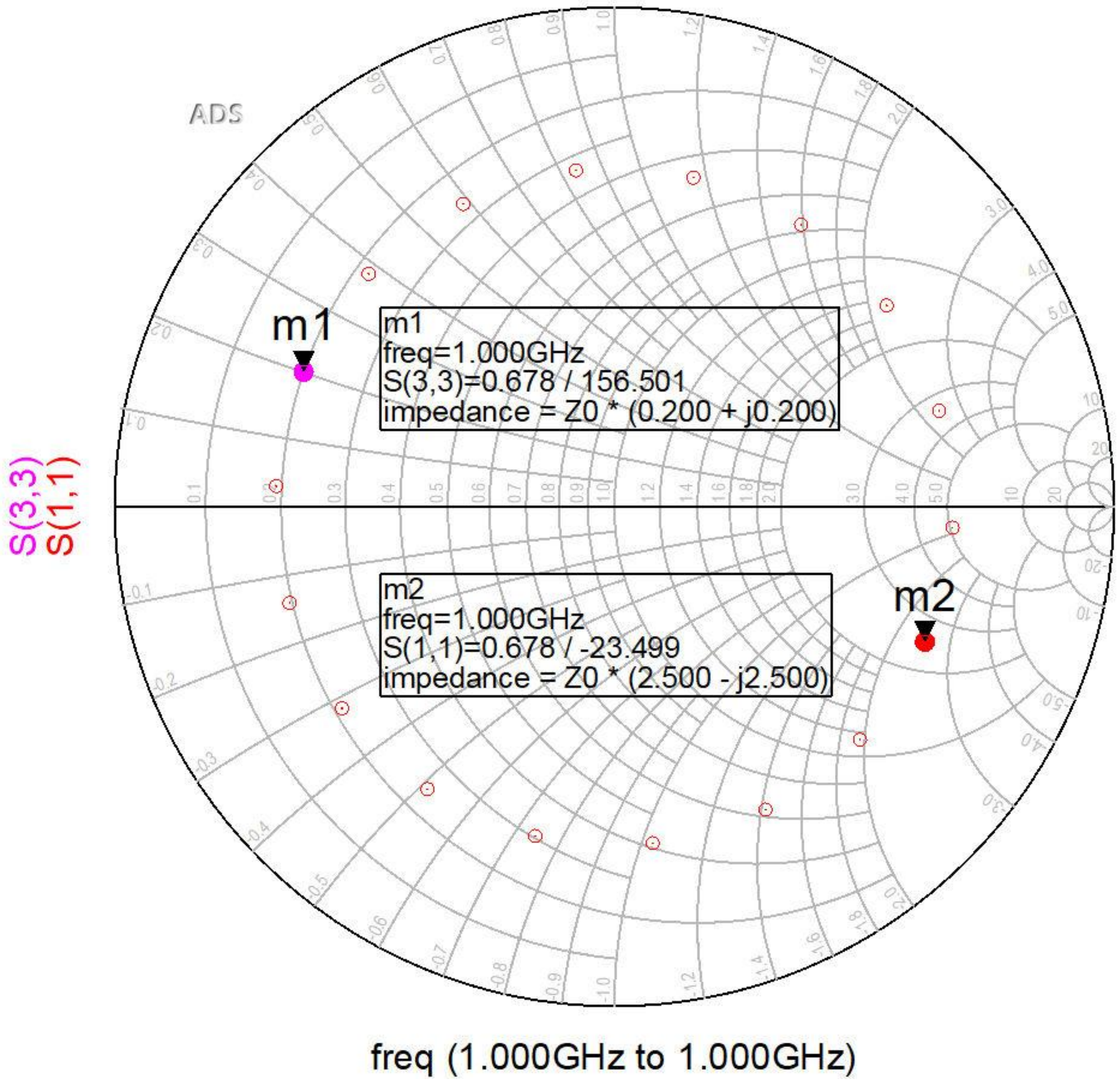
ADS, Smith Chart, series transmission line



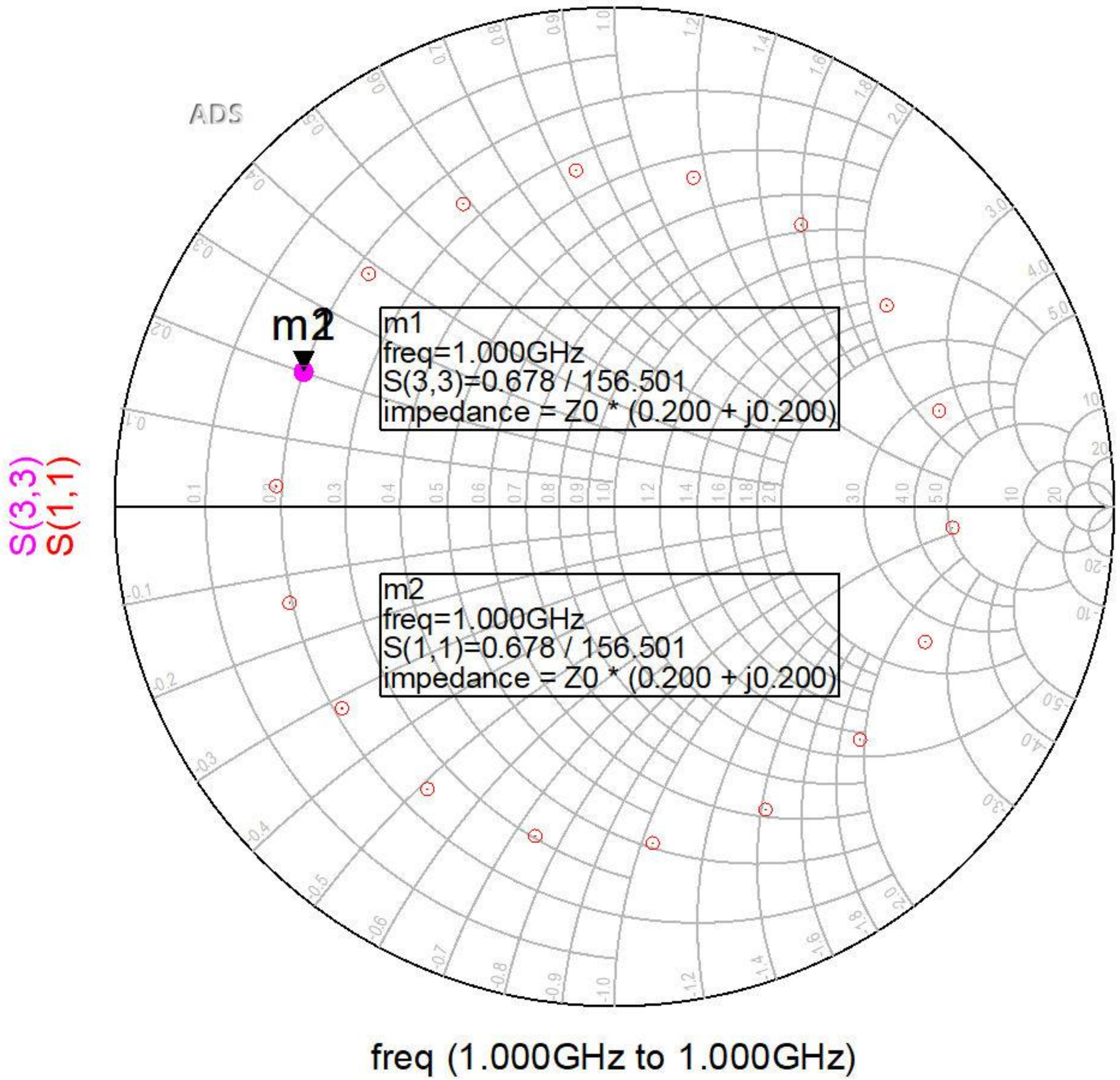
ADS, Smith Chart, series transmission line



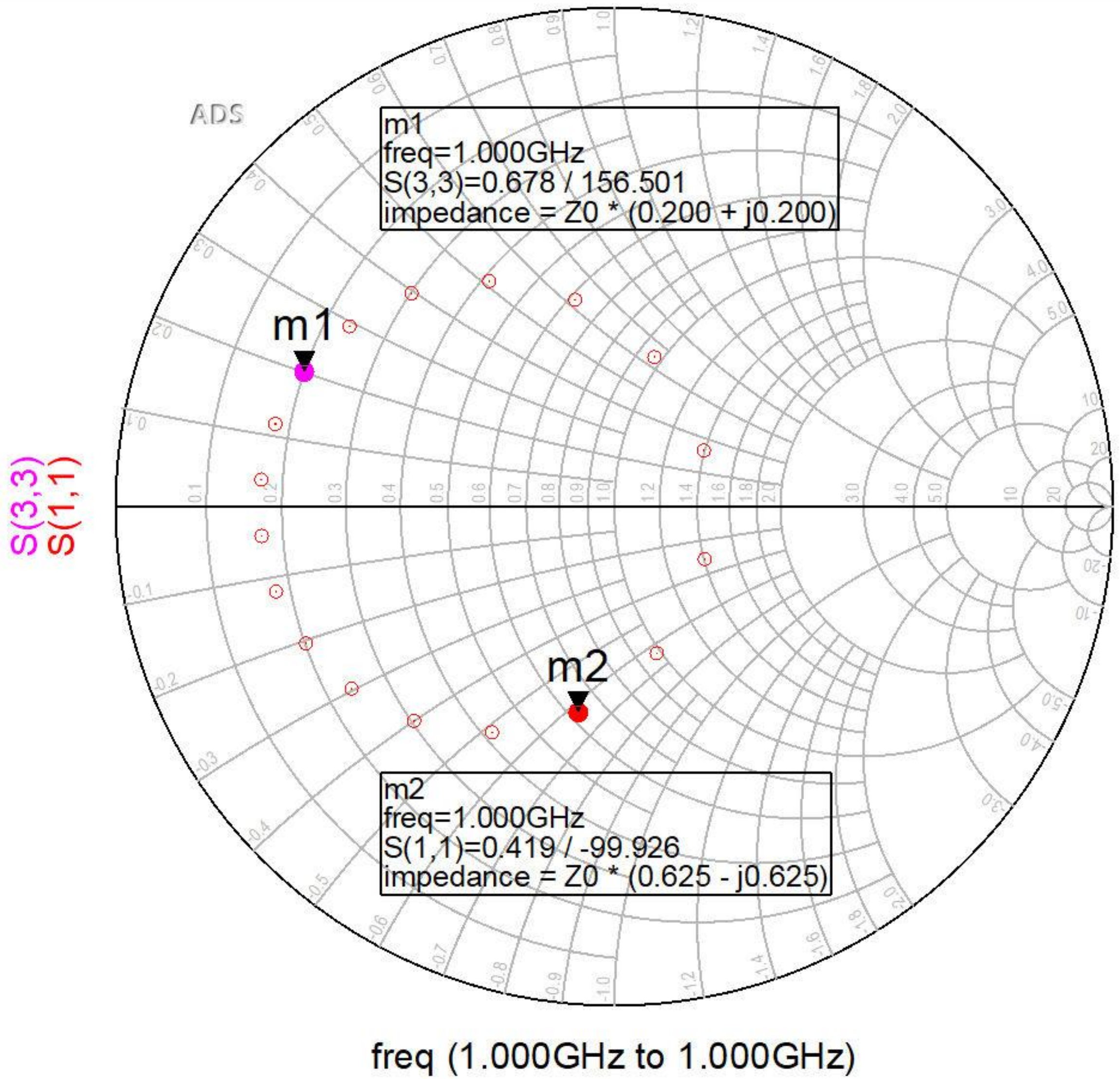
ADS, Smith Chart, series transmission line, E=90°



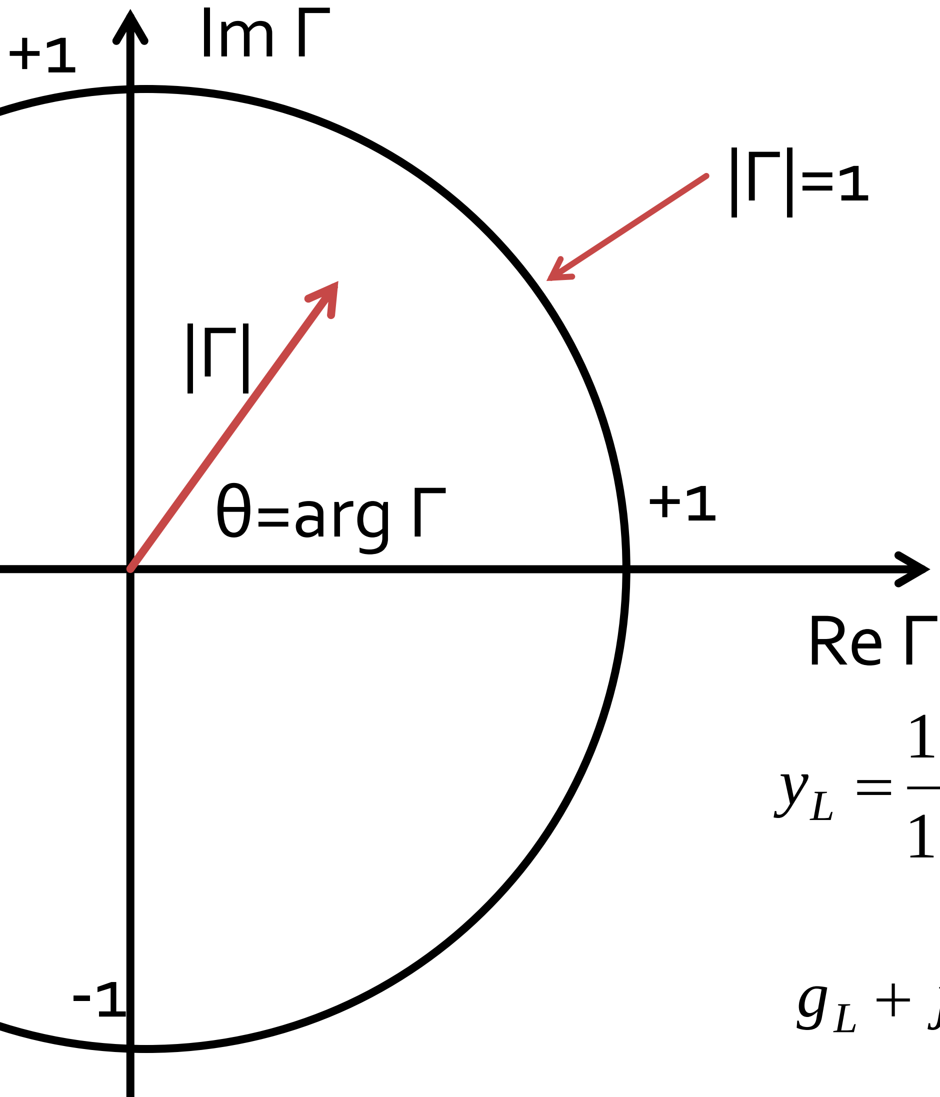
ADS, Smith Chart, series transmission line, E=180°



ADS, Smith Chart, series transmission line, Z=25Ω≠Z₀



The Admittance Smith Chart



$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{z_L - 1}{z_L + 1} = |\Gamma| \cdot e^{j\theta}$$

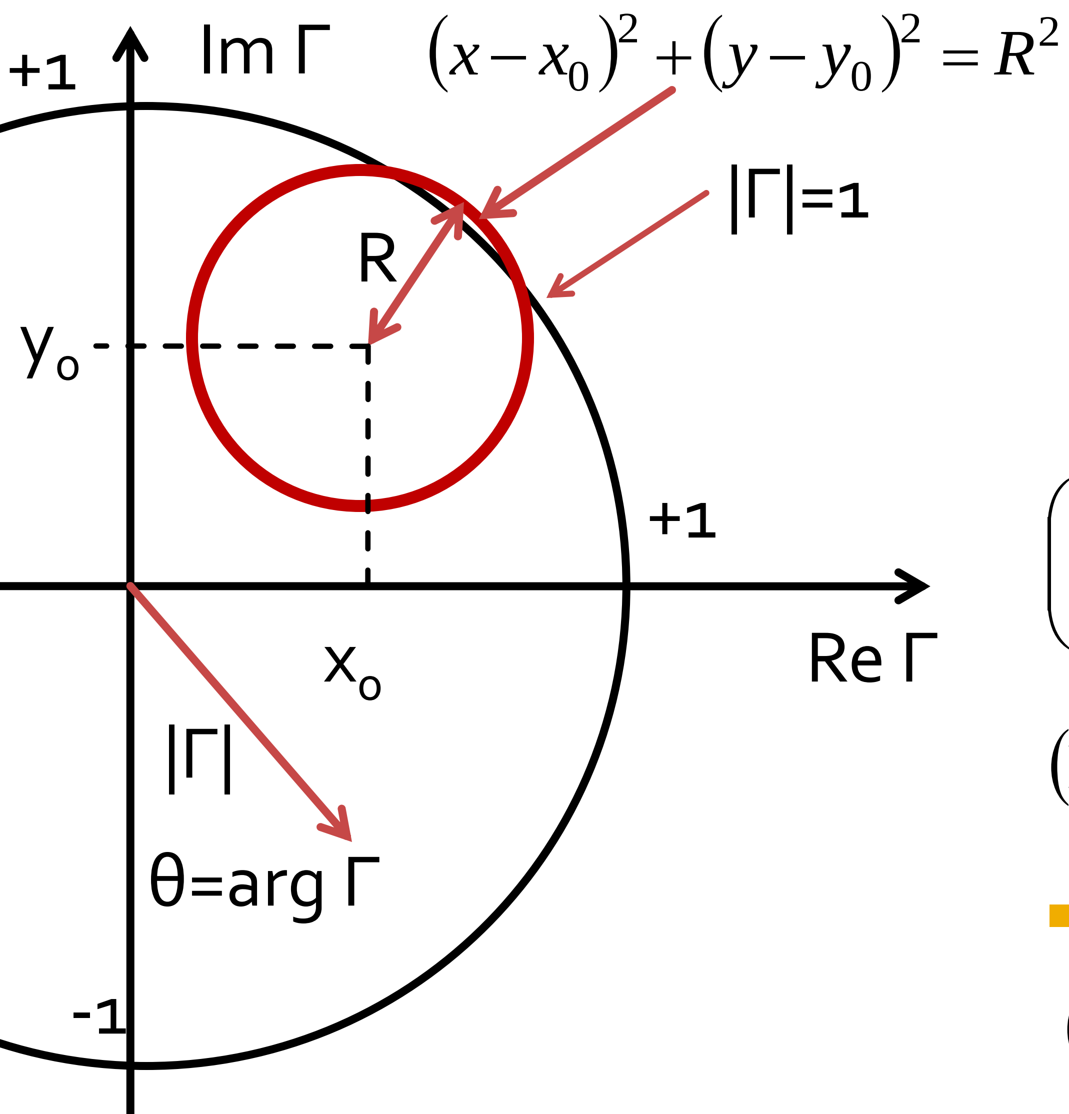
$$\Gamma = \Gamma_r + j \cdot \Gamma_i$$

$$z_L = \frac{1 + |\Gamma| \cdot e^{j\theta}}{1 - |\Gamma| \cdot e^{j\theta}} = r_L + j \cdot x_L$$

$$y_L = \frac{1 - |\Gamma| \cdot e^{j\theta}}{1 + |\Gamma| \cdot e^{j\theta}} = \frac{1}{r_L + j \cdot x_L} = g_L + j \cdot b_L$$

$$g_L + j \cdot b_L = \frac{(1 - \Gamma_r) - j \cdot \Gamma_i}{(1 + \Gamma_r) + j \cdot \Gamma_i}$$

The Admittance Smith Chart



$$g_L=\frac{1-\Gamma_r^2-\Gamma_i^2}{(1+\Gamma_r)^2+\Gamma_i^2}$$

$$b_L=\frac{-2\cdot\Gamma_i}{(1+\Gamma_r)^2+\Gamma_i^2}$$

- Rearajate

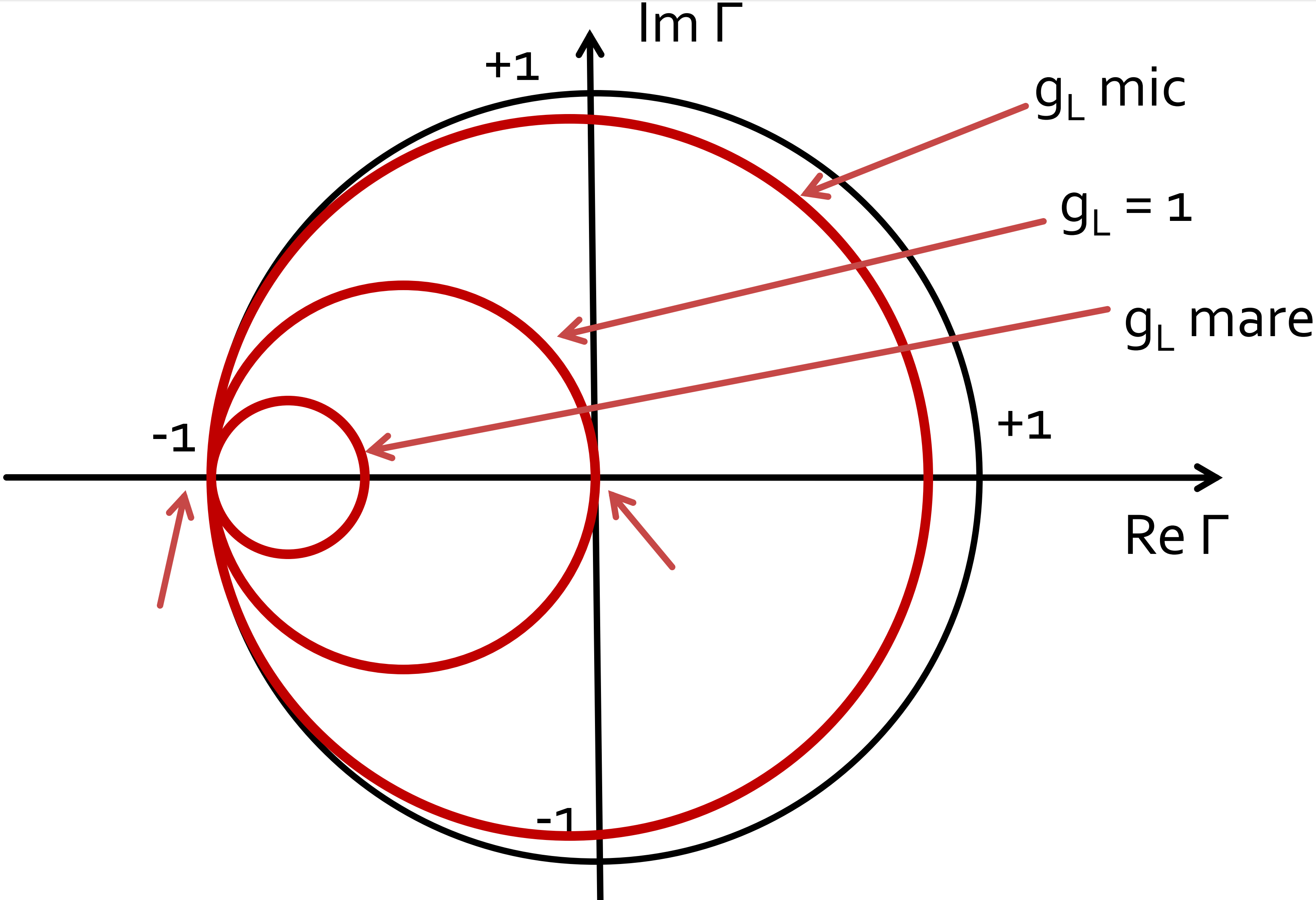
$$\left(\Gamma_r+\frac{g_L}{1+g_L}\right)^2+\Gamma_i^2=\left(\frac{1}{1+g_L}\right)^2$$

$$(\Gamma_r+1)^2+\left(\Gamma_i+\frac{1}{b_L}\right)^2=\left(\frac{1}{b_L}\right)^2$$

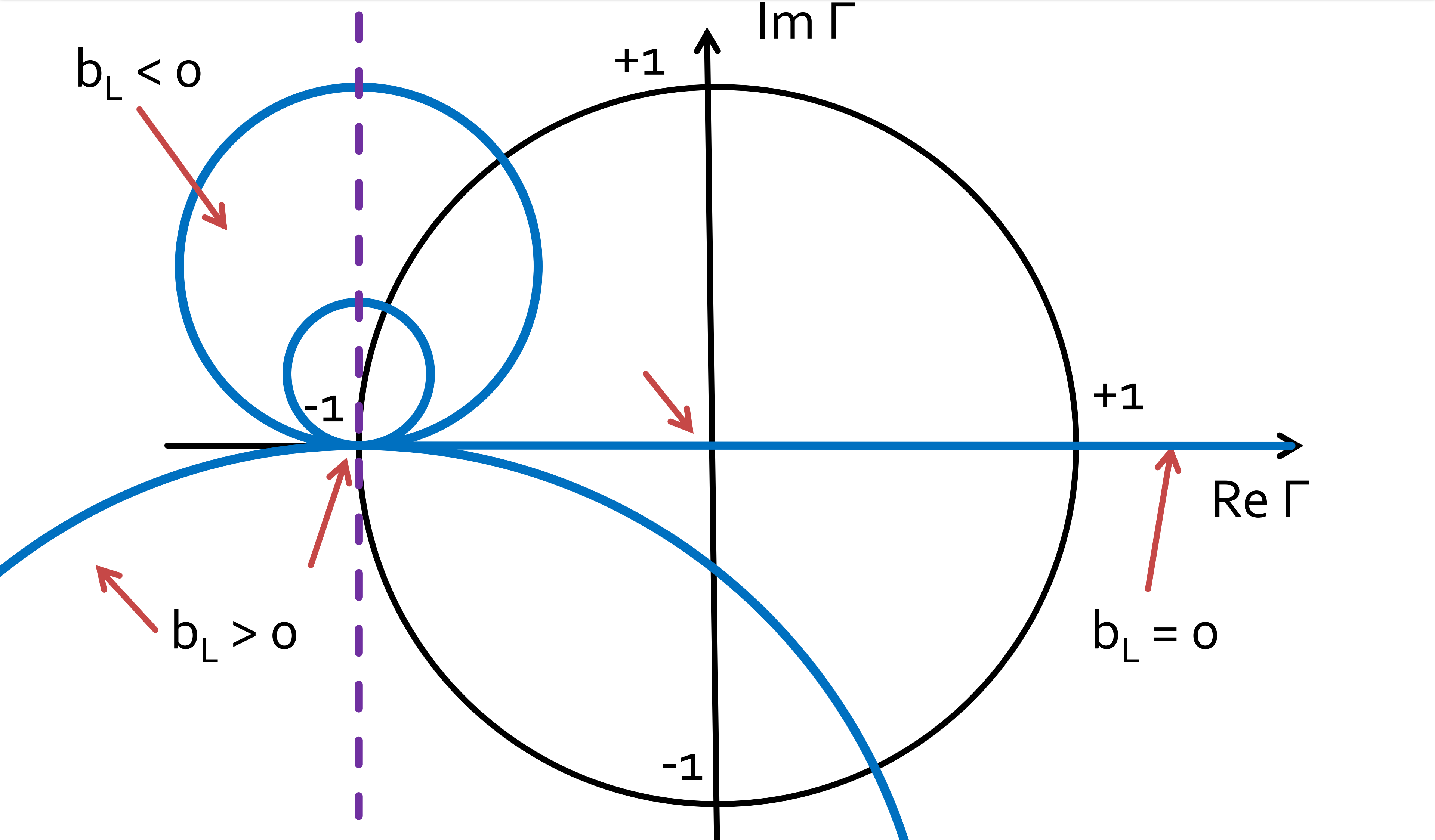
- Cercuri in planul complex

$$(x-x_0)^2+(y-y_0)^2=R^2$$

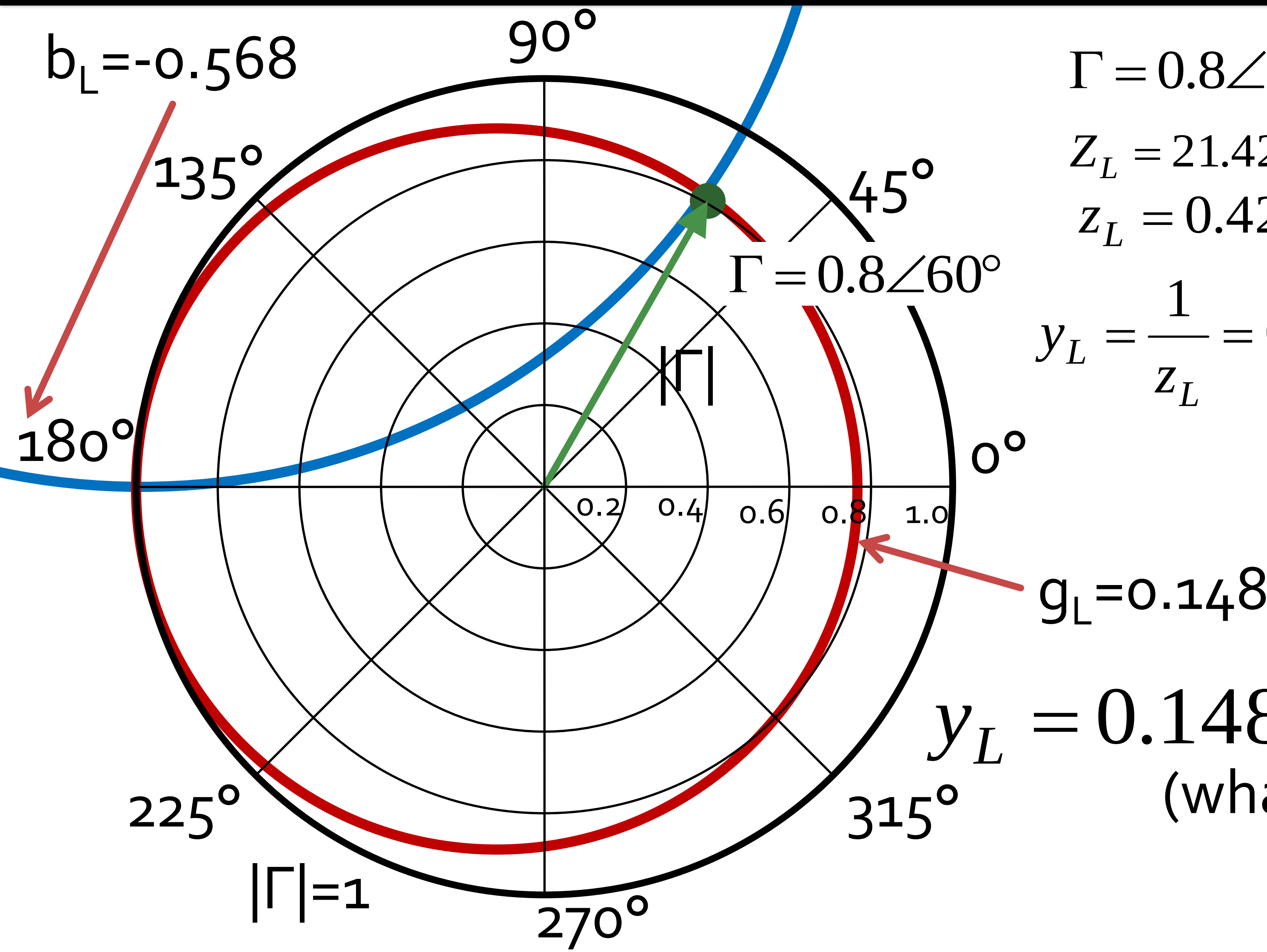
The Smith Chart, conductance



The Smith Chart, susceptance



The Smith Chart, reflection coefficient ⇔ admittance



$$\Gamma = 0.8\angle 60°$$

$$Z_L = 21.429\Omega + j\cdot 82.479\Omega$$

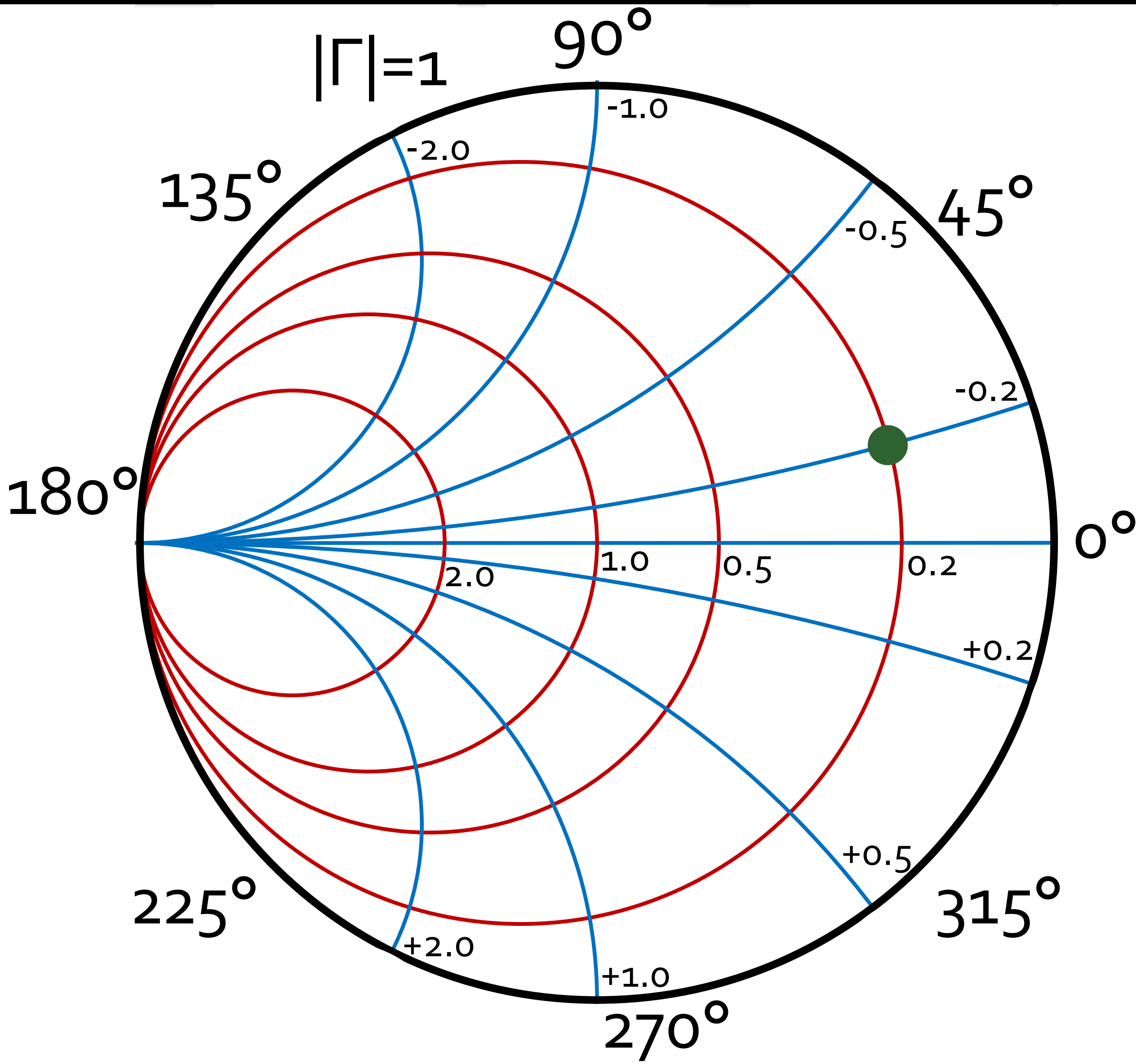
$$z_L = 0.429 + j\cdot 1.65$$

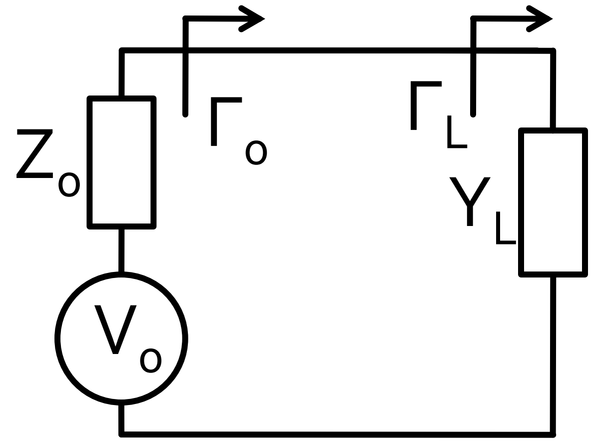
$$y_L = \frac{1}{z_L} = 0.148 - j\cdot 0.568$$

$$y_L = 0.148 + j\cdot 0.568$$

(whatever Z_o)

The Smith Chart, reflection coefficient ⇔ admittance





$$Z_0 = 50\Omega, Y_0 = 0.02S$$

$$Z_L = 125\Omega + j \cdot 125\Omega$$

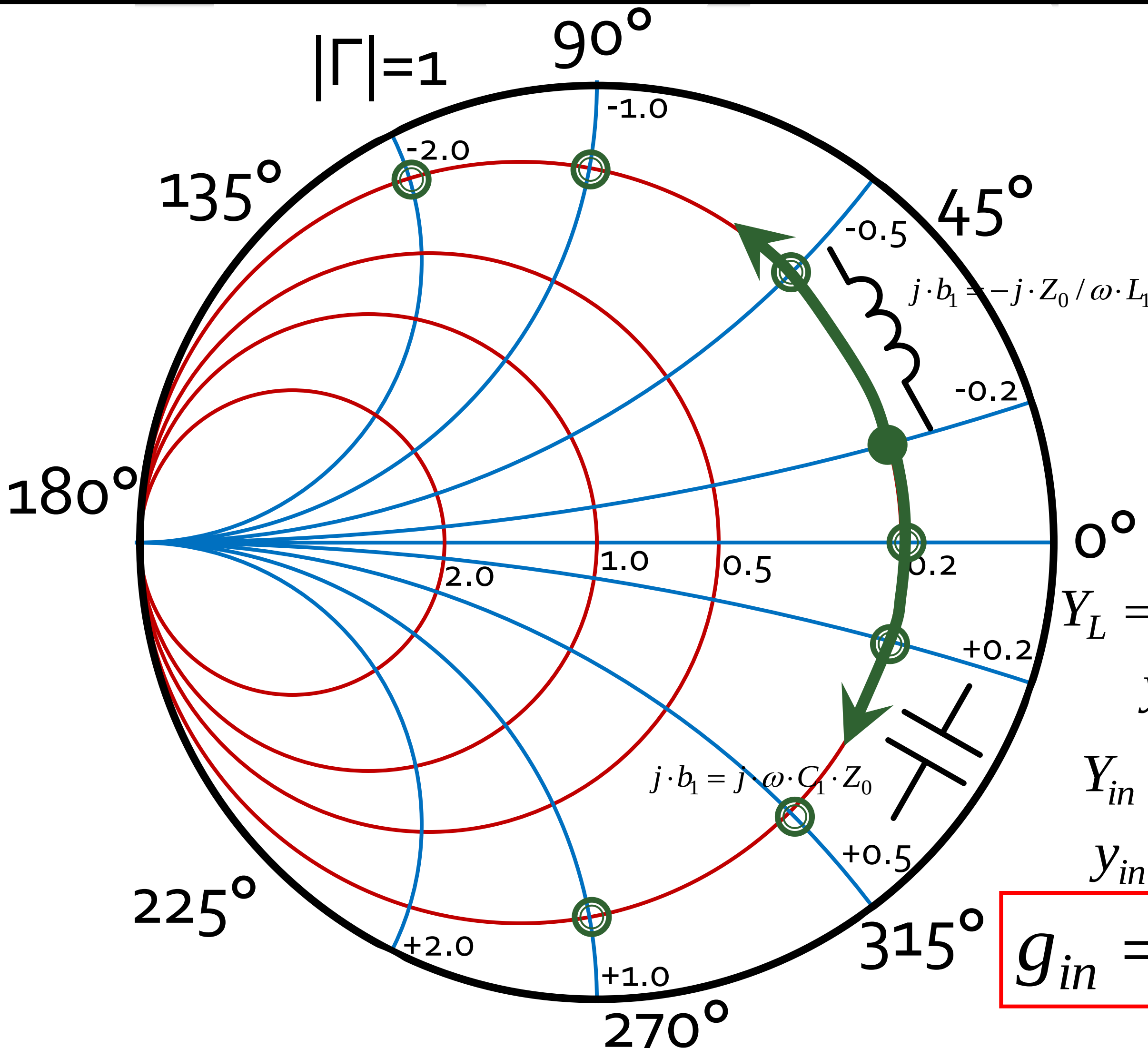
$$z_L = 2.5 + j \cdot 2.5$$

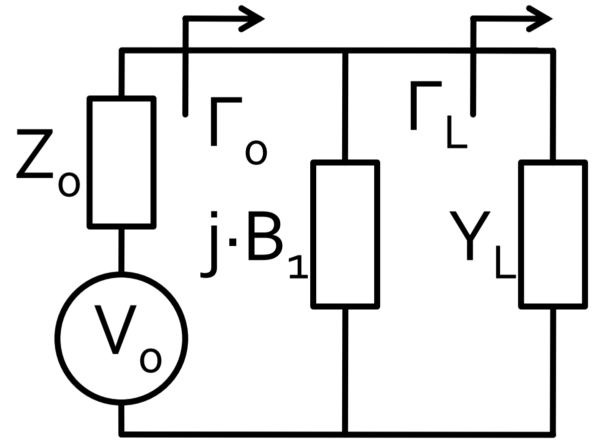
$$\Gamma_L = \Gamma_0 = 0.678\angle 23.5^\circ$$

$$Y_L = \frac{1}{Z_L} = 0.004S - j \cdot 0.004S$$

$$y_L = \frac{1}{z_L} = \frac{Y_L}{Y_0} = 0.2 - j \cdot 0.2$$

The Smith Chart, shunt susceptance





$$Z_0 = 50\Omega, Y_0 = 0.02S$$

$$\Gamma_L = 0.678\angle 23.5°$$

$$Y_L = G_L + j \cdot B_L = 0.004S + j \cdot 0.004$$

$$y_L = g_L + j \cdot b_L = 0.2 - j \cdot 0.2$$

$$Y_{in} = Y_L + j \cdot B_1 = G_L + j \cdot (B_L + B_1)$$

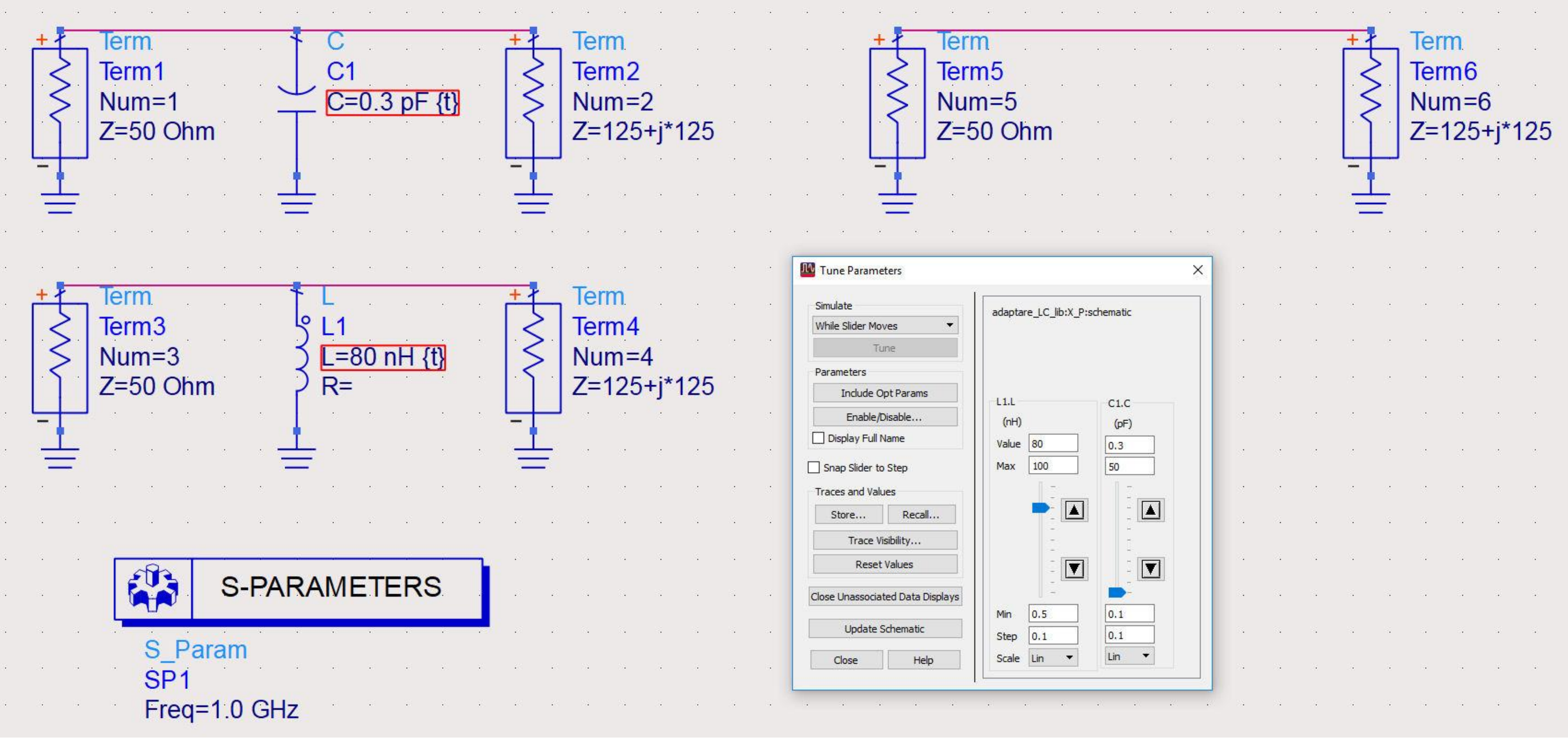
$$y_{in} = g_L + j \cdot (b_L + b_1)$$

$$g_{in} = g_L$$

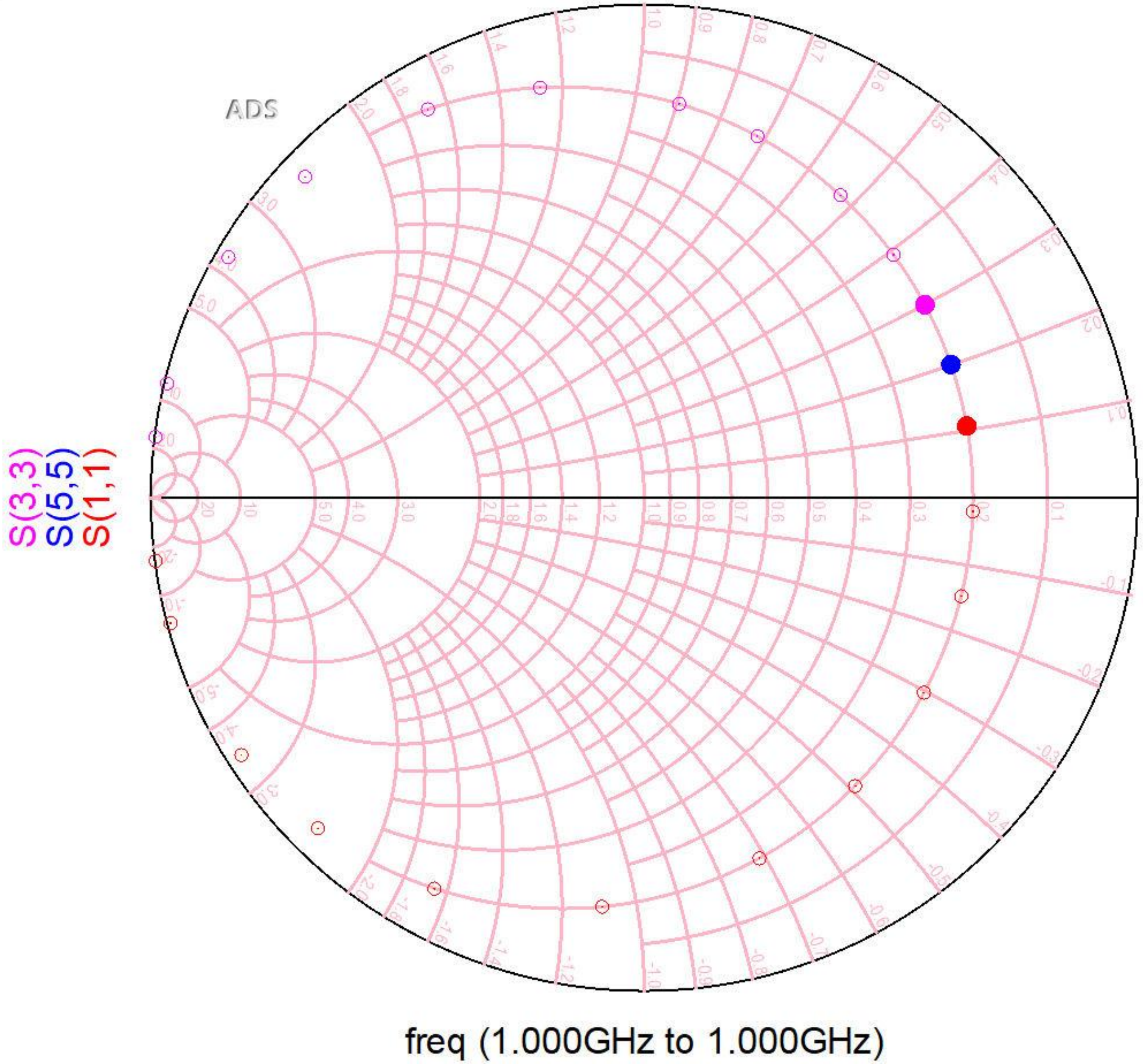
$$j \cdot b_1 = j \cdot \omega \cdot C_1 \cdot Z_0 > 0$$

$$j \cdot b_1 = -j \cdot Z_0 / \omega \cdot L_1 < 0$$

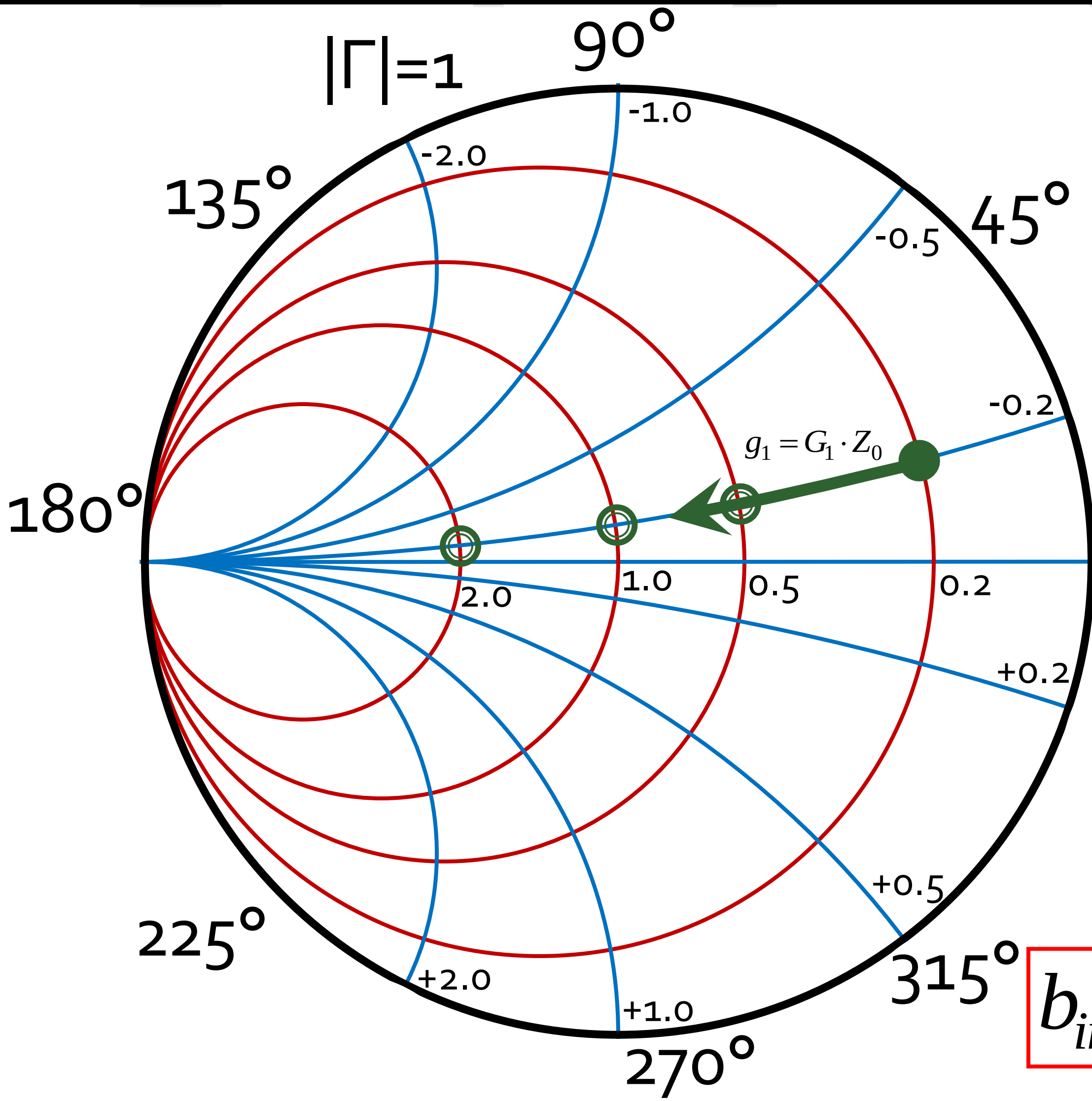
ADS, shunt susceptance

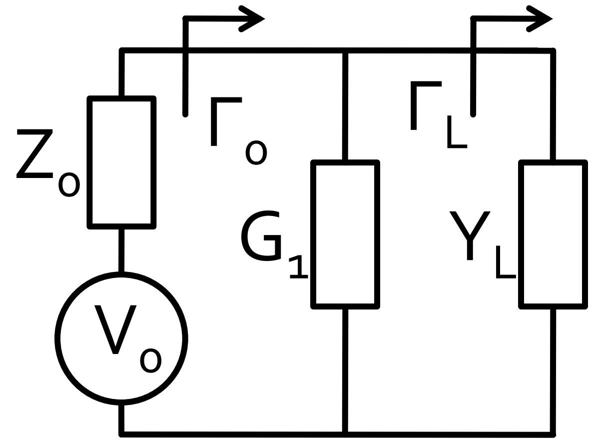


ADS, shunt susceptance



The Smith Chart, shunt conductance





$$Z_0 = 50\Omega, Y_0 = 0.02S$$

$$\Gamma_L = 0.678\angle 23.5°$$

$$Y_L = G_L + j \cdot B_L = 0.004S + j \cdot 0.004$$

$$y_L = g_L + j \cdot b_L = 0.2 - j \cdot 0.2$$

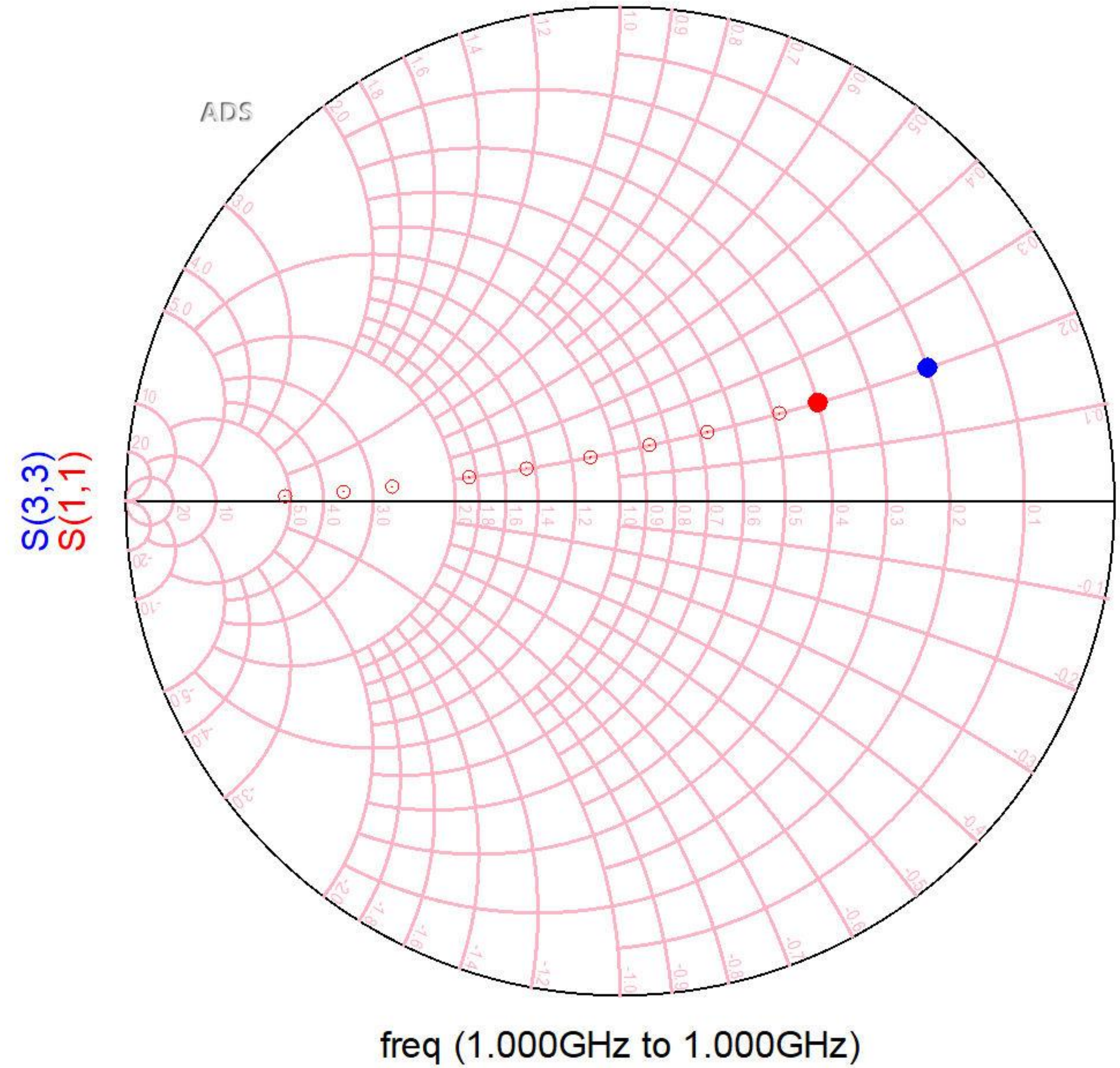
$$Y_{in} = Y_L + G_1 = (G_L + G_1) + j \cdot B_L$$

$$y_{in} = (g_L + g_1) + j \cdot b_L$$

$$b_{in} = b_L$$

$$g_{in} = g_L + G_1 \cdot Z_0$$

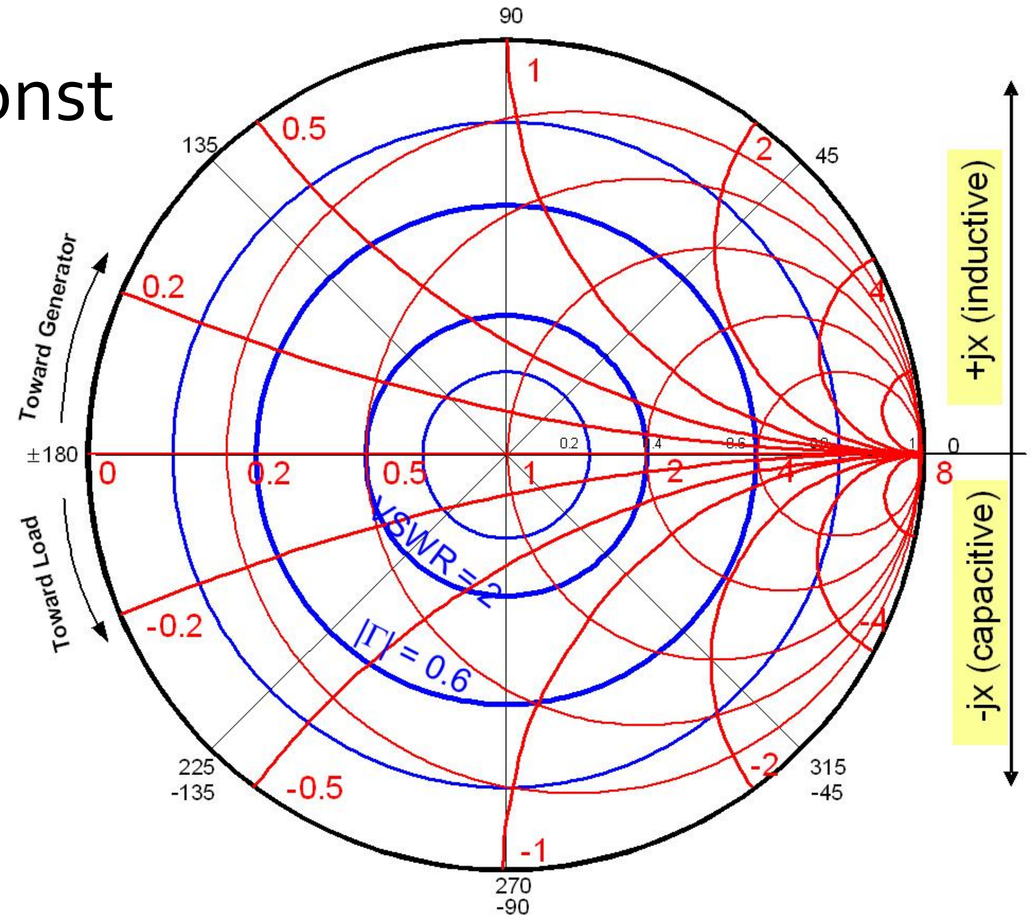
ADS, shunt conductance



Constant VSWR circles

- Certain applications may require a certain ratio between maximum / minimum line voltage
  - VSWR = const → Γ = const

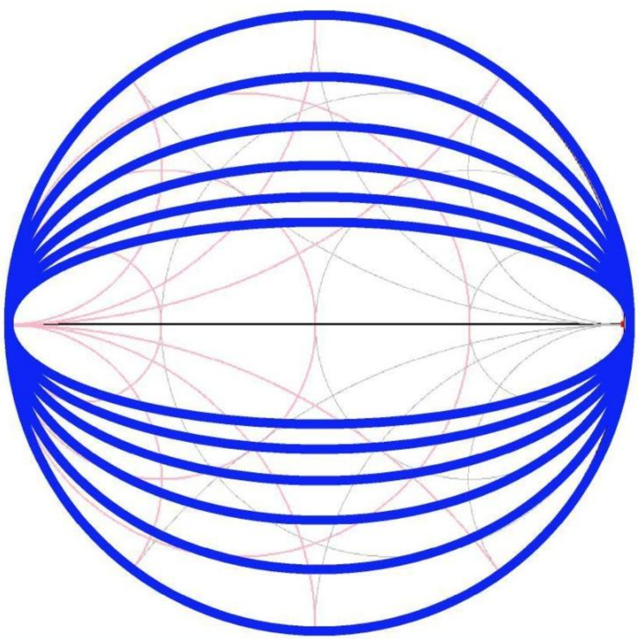
$$VSWR = \frac{V_{\max}}{V_{\min}} = \frac{1+|\Gamma|}{1-|\Gamma|}$$

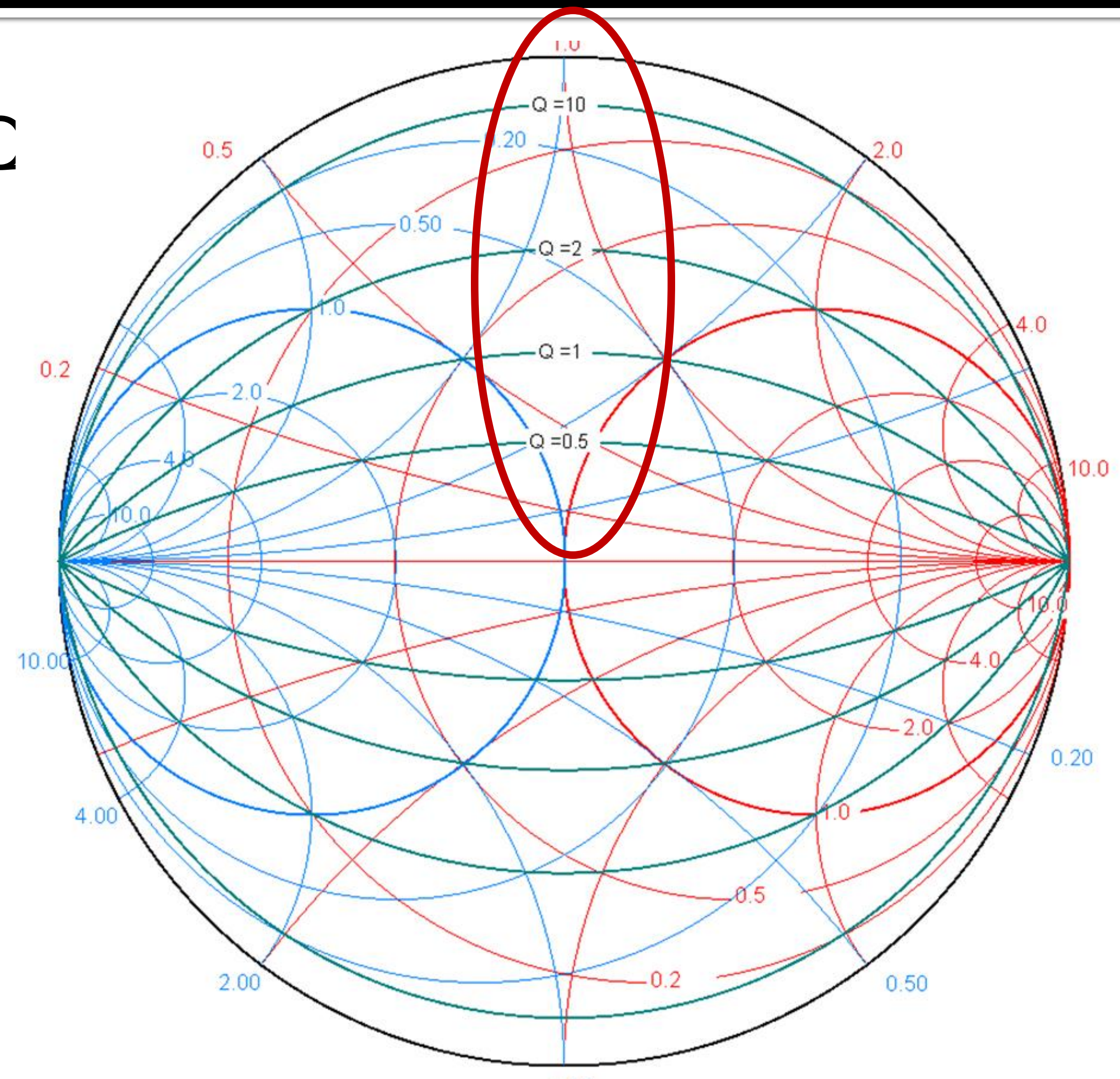


Constant Q circles

- Quality factor C

$$Q = \frac{X}{R} = \frac{G}{B} = const$$





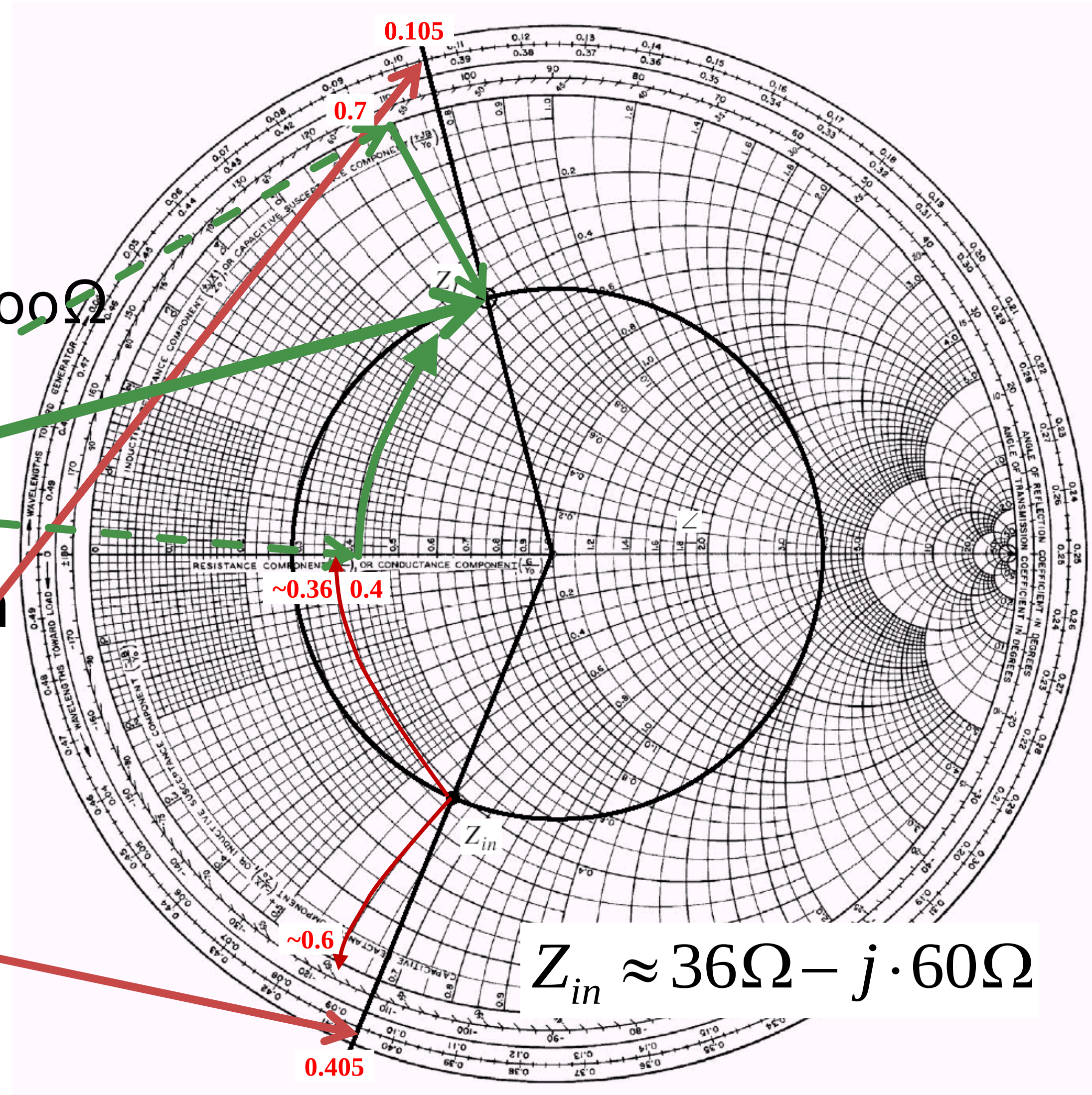
Traditional usage

- transmission line
  - 100Ω impedance
  - 0.3λ length
  - $Z_L = 40\Omega + j\cdot 70\Omega$ load
- normalization with $Z_0 = 100\Omega$

$$z_L = \frac{Z_L}{Z_0} = 0.4 + j\cdot 0.7$$

- movement with 0.3λ on a line with $Z_0 = 100\Omega$ (**circle**)
  - from z_L (0.105λ)
  - to z_{in} (0.405λ)

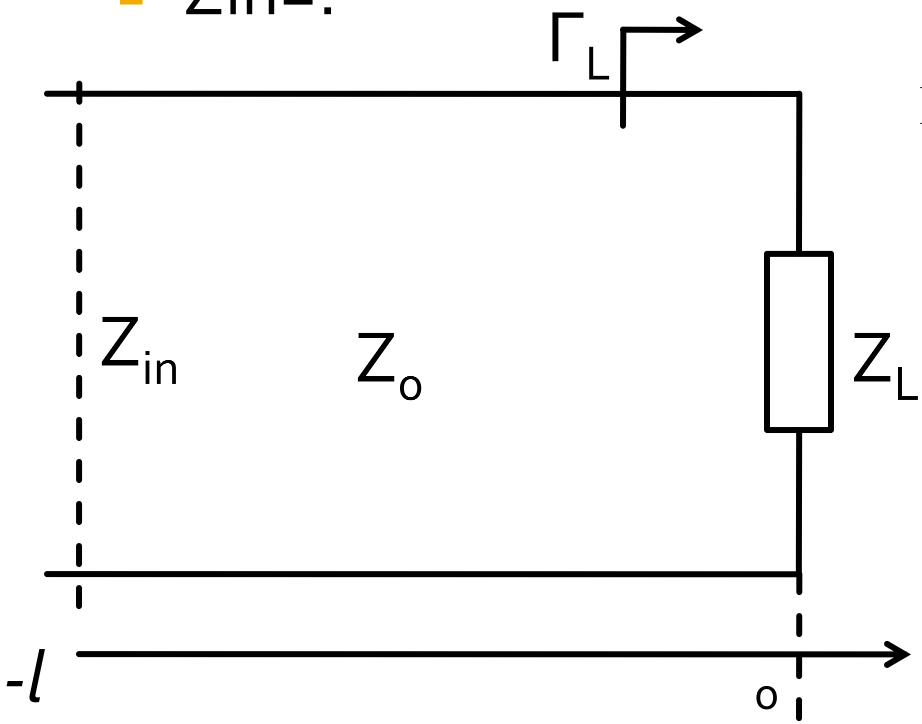
$$z_{in} \approx 0.36 - j\cdot 0.6 = \frac{Z_{in}}{Z_0}$$



$$Z_{in} \approx 36\Omega - j\cdot 60\Omega$$

Reflection coefficients

- transmission line
  - 100Ω characteristic impedance
  - 0.3λ length
  - Z_L = 40Ω+j·70Ω load
- Zin=?



$$z_L = \frac{Z_L}{Z_0} = 0.4 + j\cdot 0.7$$

$$\Gamma_L = \frac{z_L - 1}{z_L + 1} = -0.143 + j\cdot 0.571$$

$$\Gamma_{in} = \Gamma_L \cdot e^{-2j\cdot\beta\cdot l} = \Gamma_L \cdot e^{-2j\cdot\frac{2\cdot\pi}{\lambda}\cdot 0.3\lambda} = \Gamma_L \cdot e^{-1.2j\cdot\pi}$$

$$\theta = -1.2\cdot\pi = -216^\circ$$

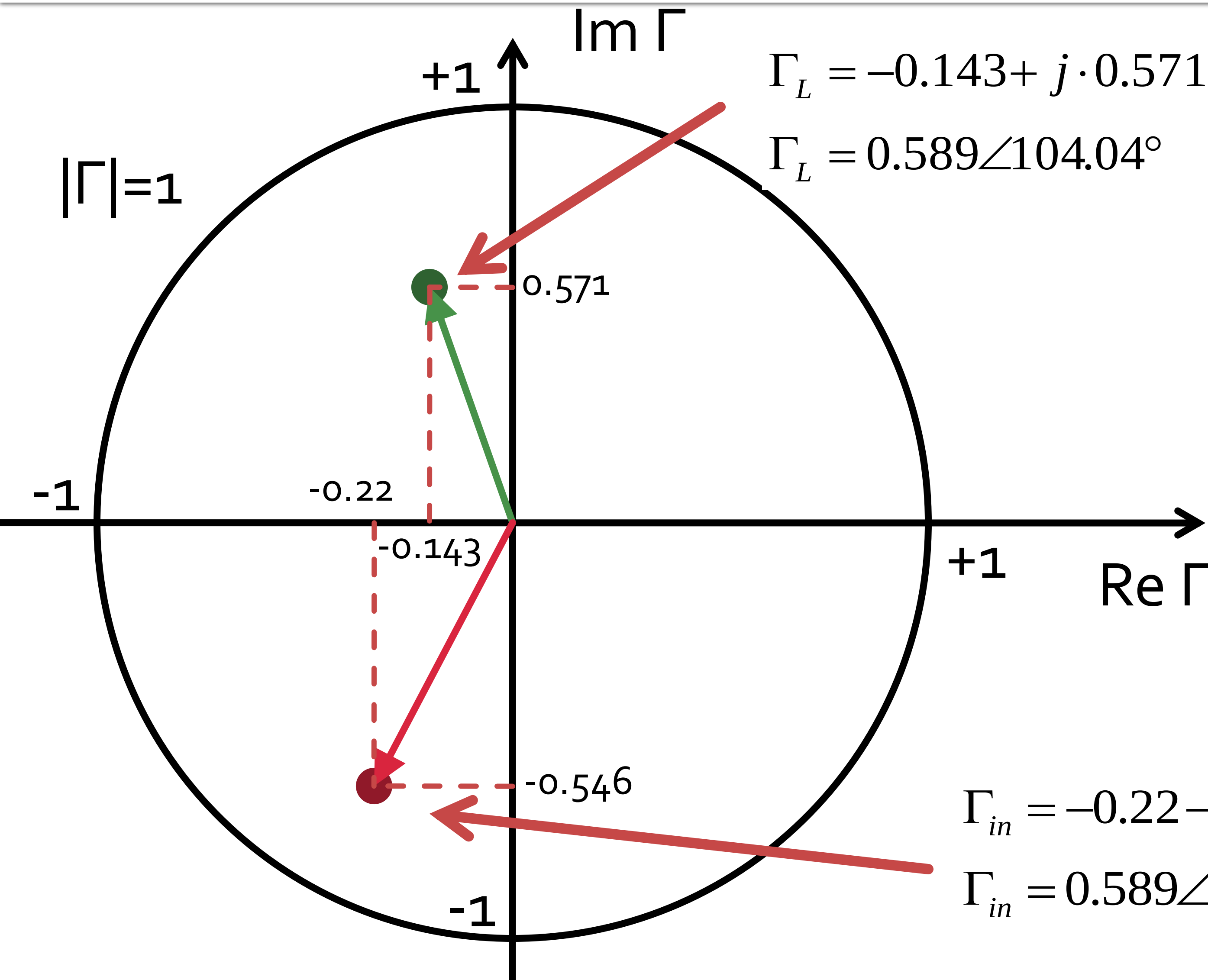
$$\Gamma_{in} = \Gamma_L \cdot \left(\cos 216^\circ - j\cdot\sin 216^\circ\right) =$$

$$= -0.22 - j\cdot 0.546$$

$$z_{in} = \frac{1+\Gamma_{in}}{1-\Gamma_{in}} = 0.365 - j\cdot 0.611$$

$$Z_{in} = z_{in}\cdot Z_0 = 36.534\,\Omega - j\cdot 61.119\,\Omega$$

Reflection coefficients



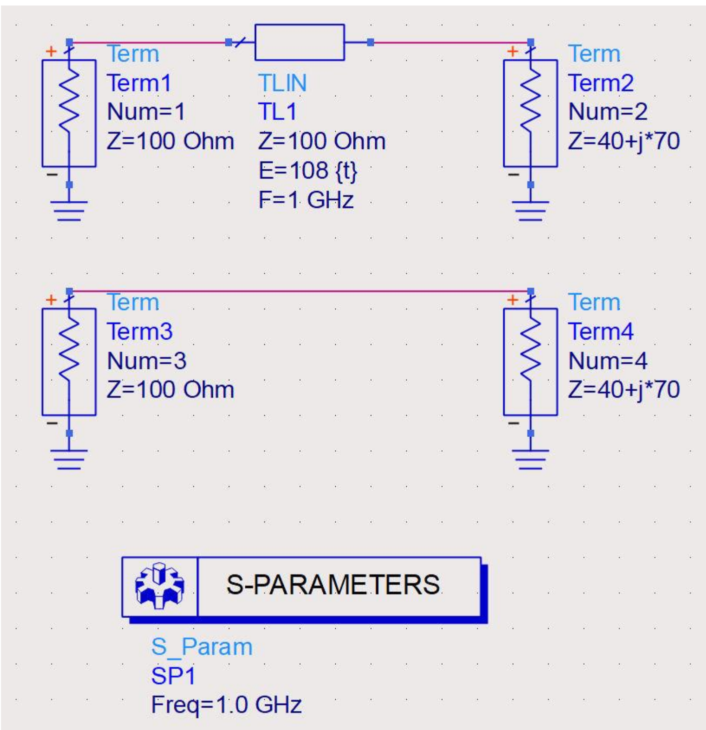
$$\Gamma_L = -0.143 + j \cdot 0.571$$

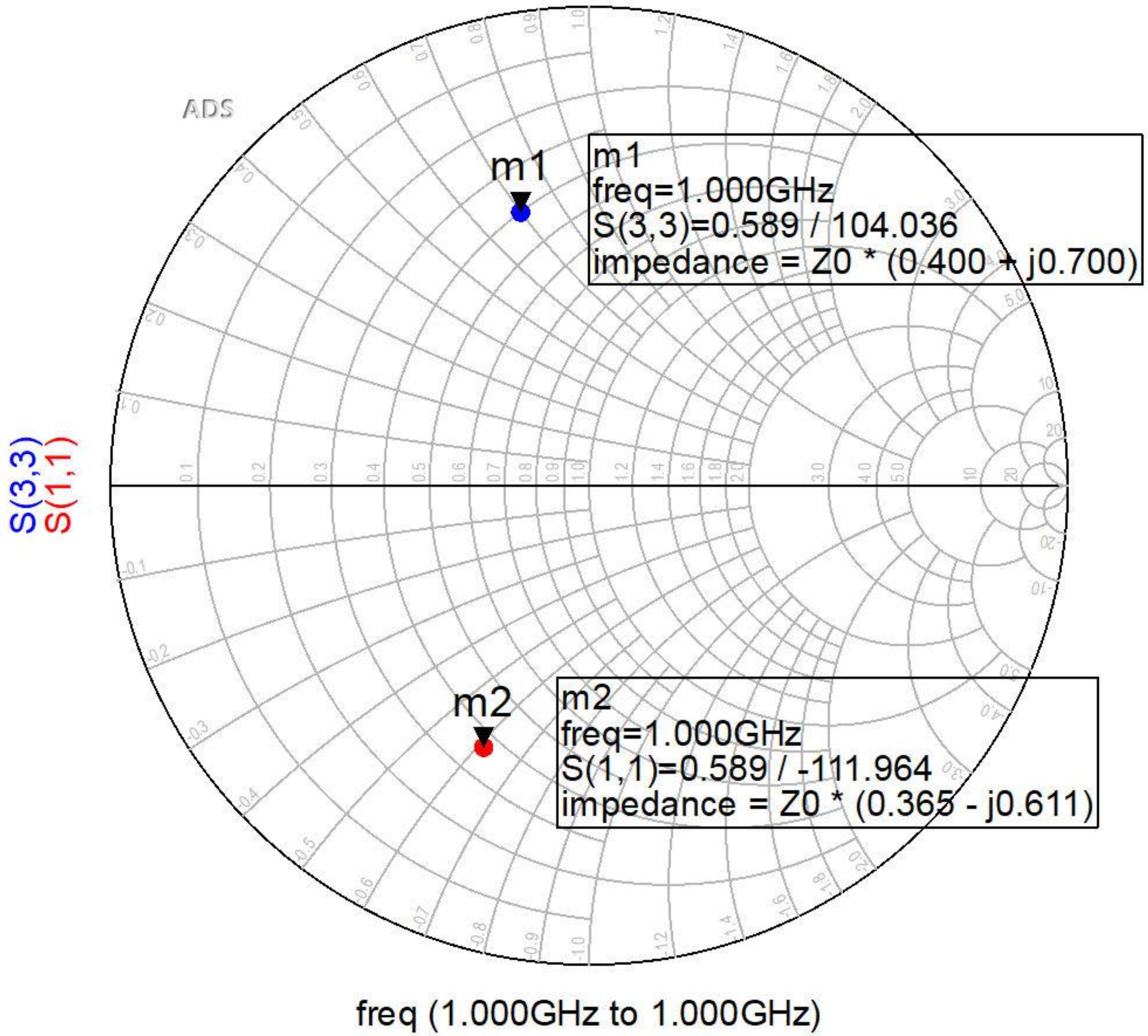
$$\Gamma_L = 0.589\angle 104.04^\circ$$

$$\Gamma_{in} = -0.22 - j \cdot 0.546$$

$$\Gamma_{in} = 0.589\angle -111.96^\circ$$

ADS, simulation





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